Next Generation Propulsion System Architectures

M. Raghavan[1*], N. Bucknor[1], J. Maguire[2], J. Hendrickson[2] and T. Singh[2]
[1] Powertrain Systems Research Lab, GM R&D Center, Michigan, USA
[2] GM Powertrain, Michigan, USA

[*] Corresponding author (email: madhu.raghavan@gm.com)

Abstract

This paper describes the creation of novel propulsion system architectures using algebraic design techniques. The objective is to create novel arrangements comprised of one or more planetary gear sets, an internal combustion engine, a pair of motor-generators and several torque-transmitting mechanisms (clutches and brakes). The algebraic design procedure represents the planetary gear sets, fixed interconnections, clutches/brakes, and motor-generator sets as algebraic constraints. Appropriate subsets of constraint equations are solved to identify viable propulsion system designs. We have used the above design approach to create several novel candidate multi-speed transmissions as well as electrically-variable transmission concepts.

Keywords: Hybrids, Transmissions, Fuel Economy

1 Introduction

A vehicle transmission delivers mechanical power from an engine to the drive system, such as fixed final drive gearing, axles and wheels. A mechanical transmission allows some freedom in engine operation, usually through alternate selection of five or six different drive ratios, a neutral selection that allows the engine to operate accessories with the vehicle stationary, and clutches or a torque converter for smooth transitions between driving ratios and to start the vehicle from rest with the engine turning. Transmission gear selection typically allows power from the engine to be delivered to the rest of the drive system with a ratio of torque multiplication/reduction and with a reverse ratio.

An electrically-variable transmission (EVT) is a mechanical transmission augmented by one or more electric motor/generators. A motor/generator is a device, which uses battery power to apply a torque on a transmission member (in motor mode), or generates power for storage in the battery, while serving as a speed-controlled brake (in generator mode). Typically, an EVT uses differential gearing to send a fraction of its transmitted power through a pair of motor/generators. The remainder of its power flows through another, parallel path that is all mechanical and direct, of fixed ratio, or alternatively selectable. One form of differential gearing is the well-known planetary gear set with the advantages of compactness and different torque and speed ratios among the various members of the gear set. Electric energy storage in the battery allows the mechanical output power from the transmission system to the vehicle, to vary from the mechanical input power from the engine to the transmission system. The battery or other device also allows for engine starting with the transmission system and for regenerative vehicle braking.

An electrically variable transmission in a vehicle can simply transmit mechanical power from an engine input to a final drive output. To do so, the electric power produced by one motor/generator balances the electrical losses and the electric power consumed by the other motor/generator. By using the electrical storage battery, the electric power generated by one motor/generator can be greater than or less than the electric power consumed by the other. Electric power from the battery can sometimes allow both motor/generators to act as motors, especially to assist the engine with vehicle acceleration. Both motors can sometimes act as generators to recharge the battery, especially in regenerative vehicle braking.

2 Background and Prior Work

There is considerable interest in novel transmissions with a high ratio-spread and a large number of speeds. The main benefits from such designs are improved performance, better fuel economy, and smoother shifts. The increased number of speeds reduces the step size between ratios, improving shift quality by making ratio interchanges substantially imperceptible to the driver. Improved fuel economy is obtained because the large number of speed ratios allows the engine to operate at or near its optimum operating point for most of the time. There is also the promise of improved brand equity via the marketing of advanced technological features. Much of the understanding of multi-speed transmission kinematic operation has been described in the language of lever diagrams [1]. Since the automobile industry is generally moving in the direction of larger numbers of fixed speed ratios we briefly review recent work on multi-speed transmissions.

Haka [2] proposes a design with 3 planetary gear sets. One gear set is a dedicated gear set in that one of the planetary members (reaction member) is permanently connected to a stationary member and another is continually connected to the input drive member. The dedicated planetary gear set can be arranged to provide either an underdrive or an overdrive depending on the input and reaction members. The resulting design has at least 7 forward drive ratios and one reverse drive ratio. Borgerson et al., [3] present the design of a six-speed transmission having an input shaft connectable with an engine and planetary gear unit. A single carrier supports pinions from adjacent planes of gears. Stevenson [4] presents a seven-speed concept with three planetary gear sets. The second and third planetaries are connected. The design has six torque transmitting mechanisms, engaged in sets of two to get seven forward speeds and one reverse speed ratio. Wittkopp [5] proposes a three planetary design with three brakes, three clutches, and three fixed interconnections between the gear sets.

2.1 Prior Work on Electrically Variable Transmissions

We briefly review some recent offerings in the EVT literature. Malikopoulos et al., [6], describes the development of an Integrated Starter Alternator (ISA) for a High Mobility Multi-Purpose Wheeled Vehicle. Its primary purpose is to provide electric power for additional accessories but it can also be used for mild hybridization of the powertrain. Simulation studies show that the ISA powertrain can improve fuel economy by 4.3% over a combined city-highway driving cycle. Pagerit et al., [7] study several vehicle platform and powertrain configurations to assess the sensitivity of fuel economy to mass variation. Their conclusion is that conventional and parallel hybrid configurations are the most sensitive while fuel cell-based arrangements are the least sensitive. Suppes [8] argues that closed-system regenerative fuel cells (RFCs) are an alternative to non-regenerative fuel cells as a transition technology and mainstay of a hydrogen economy. He suggests that substantially petroleum-free automobiles can evolve from hybrid electric vehicles as fuel cell prices decrease. The evolution can be projected first to plug-in hybrid electric vehicles and finally to a substantially hydrogen-based transportation system.

Tamai et al., [9], review the essential features of the hybrid system for the 2007 Model Year Saturn VUE Green Line Hybrid SUV. This concept provides the fuel economy of a compact sedan while delivering improved acceleration performance over the base vehicle. Key elements of the powertrain are a 2.4L DOHC engine with dual cam-phasers, a modified 4-speed automatic transmission, an electric motor-generator connected to the crankshaft through a bi-directional belt-drive system, power electronics to control the motor-generator, and a NiMH battery pack. The VUE's hybrid functionality includes: engine stop-start, regenerative braking, intelligent charge control of the hybrid battery, electric power assist, and electrically motored creep.

An example of a highly successful EVT concept developed at GM is the two-range, input-split and compound-split electrically variable transmission now produced for transit buses. This EVT was invented at Allison Transmission by Schmidt [10]. One embodiment of this idea employs three planetary gear sets coaxially aligned. The two motor/generator sets are also coaxially aligned with the planetary gear sets. Gear members of the first and second planetary gear set are respectively connected to the two motor/generators. Their carriers are operatively connected to the output member. Today's typical single-mode systems rely on much larger electric motors than are needed in the patent-protected two-mode system. The two-mode system innovations provide performance and fuel economy improvements at highway speeds and better trailer towing ability. Packaging is more efficient than today's single mode designs as the system's compact and powerful electric motors are designed to fit within the approximate space of a conventional automatic transmission. This system reduces fuel consumption at highway speeds much more effectively than available single mode systems and achieves at least a 25 percent improvement in composite fuel economy in full-size truck applications.

Table 1: Sample Prior Work on Transmissions & Hybrids

Haka	*7-speed, 3 planetaries, one stationary member*
Borgerson	*6-speed, input selectively connected to engine*
Stevenson	*7-speed, 3 planetaries, 6 clutches/brakes*
Wittkopp	*7-speed, 3 planetaries, 3 fixed interconnections*
Malikopoulos	*Integrated Starter Alternator*
Suppes	*Closed-system regenerative fuel cell*
Tamai	*Belt Alternator Starter Hybrid*
Schmidt	*Two Mode Hybrid*

2.2 Significance of the Present Work

The main benefit of the algebraic procedure described here is that it allows the designer to systematically generate and assess novel designs. The computer process often proposes unusual arrangements, which even experienced designers might overlook. Furthermore, as the requirements on fuel economy and performance compel manufacturers to use transmissions with higher numbers of speed ratios, transmission designers have to deal with increasingly complex mechanisms. Another benefit is its ability to identify minimum-content designs, wherein the emphasis is on achieving the maximum level of functionality with the fewest components. This is particularly evident in the design of commercial truck transmissions where there is a need for multiple reverse ratios for various operating conditions, in addition to the large number of forward ratios required.

3 Generation of New Concepts

The steps of the process are described in detail by Raghavan et al. [11]. They may be summarized as follows. We make an upfront decision regarding the number of planetary gear sets and clutches to be used in the proposed transmission. For example, we may choose to investigate designs with 3 planetary gear sets and 6 or 7 friction elements in a quest for 8-speed transmission mechanisms. We enumerate all possible kinematic combinations of these elements that could potentially serve as legitimate transmission mechanisms. To do this we utilize the transmission governing equations to identify specific configurations that yield viable 8-speed designs. We then select candidates that satisfy additional requirements, such as ratio spread, step ratios, reverse-to-first ratios, single-transition shifts, etc.

There are several details to be followed in the above procedure. First we must decide on whether the input to the transmission is fixed to one of the transmission members or clutched to it. We must also decide on how many clutches/brakes we engage at any given speed ratio, as this would determine the number of constraints on the system. Typically, we select a scheme with the maximum possible number of speed ratios. This involves some combinatorics calculations. Next, we decide on the number and type of fixed interconnections between various members of the planetary gear sets. An edge-vertex representation of transmission mechanisms is most useful in this step, as it allows the sorting of designs based on graph theory [12], [13].

These decisions serve to focus our search into specific "families" of transmission mechanisms. After that, we formulate algebraic representations of the various transmission candidates, using equations to describe all of the applicable constraints, such as clutches, brakes, etc. The key enabling concepts that make this synthesis procedure work are: algebraic representation of geared kinematic systems, topological representations of mechanisms and graph isomorphism, generalized lever diagrams, which allow unified code generation and computational efficiencies, fast numerical methods to rapidly search large multi-dimensional design spaces.

The functional requirements for EVTs may be grouped into several operating modes. The first operating mode is the "battery reverse mode" in which the engine is off and the transmission element connected to the engine is not controlled by engine torque, though there may be some residual torque due to the rotational inertia of the engine. The EVT is driven by one of the motor/generators using energy from the battery, causing the vehicle to move in reverse. Depending on the kinematic configuration, the other/motor/generator may or may not rotate in this mode, and may or may not transmit torque. The second operating mode is the "EVT reverse mode" in which the EVT is driven by the engine and by one of the motor/generators. The other motor/generator operates in generator mode and transfers 100% of the generated energy back to the driving motor. The net effect is to drive the vehicle in reverse. The third operating mode includes the "reverse and forward charging modes." In this mode, the EVT is driven by the engine and one of the motor/generators. A selectable fraction of the energy generated in the generator unit is stored in the battery, with the remaining energy being transferred to the motor. The fourth operating mode is a "continuously variable transmission range mode" in which the EVT is driven by the engine as well as one of the motor/generators operating as a motor. The other motor/generator operates as a generator and transfers 100% of the generated energy back to the motor. The fifth operating mode includes the "fixed ratio" configurations in which the transmission operates like a conventional automatic transmission, with torque transfer mechanisms (clutches or brakes) engaged to create a discrete transmission ratio.

4 Graph Sorting

The above concept generation/enumeration process produces a large amount of data which must be post-processed to find valid designs. This requires interpreting the data and drawing a sketch of the transmission cross-section. With potentially millions of designs, this can be a time consuming process. There are two issues: 1) a large number of the designs are not unique because the generalized method allows many representations of the same design; 2) many of the designs, while kinematically correct, are not topologically feasible. That is, when we attempt to sketch the 2-D transmission cross section we may find that there is no way to connect all of the elements (i.e., the gear sets, clutches, fixed interconnections and shafts) without interferences between connections. Several attempts may be necessary to determine that we have exhausted all of the potential ways to draw the cross-section before deciding that it is impossible and there is some uncertainty to the decision. To achieve the best efficiency in sketching designs, it would therefore be helpful to know a priori and with certainty whether or not a design is possible.

Once we have our graph representation in hand for each synthesized design, we need to check the design for feasibility and uniqueness. As noted above, we only need to test that a graph is planar to decide whether or not the design is feasible. Fortunately, such a test (the Hopcroft-Tarjan algorithm [14]) exists and we use an implementation of it. Once feasibility is determined, we save the design and compare it to all remaining designs using a graph uniqueness (or isomorphism) test described by Tsai [15].

5 Analyses of Architecture Concepts against System Requirements and Integration Constraints

So far this work has described a design process that utilizes a process of a priori selection, enumeration, and sorting to identify transmission architecture concepts. The transmission architecture concepts identified by the process are commonly referred to as powerflows. The next phase in the design process is to analyze the various powerflows identified against high level vehicle requirements to sort out the most ideal candidate for design and implementation. The best transmission design is the one that interacts with other vehicle subsystems to ensure that the vehicle best meets its design goals. Both quantitative and qualitative measures of customer perceived quality in areas such as safety, dependability, reliability, cost, efficiency, performance, comfort, payload, excitement, quietness, quiescence, compactness, and delivery timing are evaluated for each candidate powerflow to determine which powerflow should be selected for a given portfolio of vehicle applications. The relative importance of each aspect of customer quality varies depending on market and vehicle segments. Sports vehicles emphasize performance and excitement over fuel efficiency and payload where commercial vehicles emphasize payload and efficiency over quietness and excitement.

Modern computer systems enable both vehicle level simulations and subsystem level analyses to determine the resultant conformance of each component set to various design criteria. Results from these simulations are then captured in a Pugh Concept Selection Matrix to rank the relative strengths and weakness of various design alternatives against an existing baseline. If results are favorable or if an initial design is required to obtain critical sort information then a decision may be made to pursue an initial design of a candidate powerflow.

5.1 Prioritization Process

To some extent key aspects of each powerflow have already been identified (e.g. overall ratio, ratio steps, element speed ratios) early in the process. Once these characteristics have been identified they can be used to initially quantify a powerflow's merit. This allows the rough prioritization of candidate powerflows.

This initial prioritization process does not typically identify a single superior candidate; rather several powerflows will have collectively similar results. At this point more detailed metrics can be developed that will further identify the merits of each of the top candidates. Strategic insight can be gained from the powerflow alone. These characteristics are independent of the final design and development details: relative torques on components, relative slip speed between components, kinematic content. This type of data can be used to populate a "traffic light chart" (see Fig. (1)) that will further refine the list of leading candidates.

	Powertrain Matching					Risk		Pack.	Cost	Fuel Econ.	
Power Flow	# of Spd	Top Gr	Top Step	OAR	Ratio Progression	Mechanical Tech. Level	Controls Tech. Level	Planes of Gears	Kinematic Cost Index	Spin Loss Index	Mesh Eff
Power Flow	5	0.53	1.23	5.4		3	2	2	17.5	4.8	98
PF A	6	0.69	1.23	5-6		1	2	3	19	4.6	98.85
PF B	5	0.49	1.38	5-6		1	1	2	22	11.7	98.3
PF C	6	0.64	1.24	6.22		3	3	3	19		96.7
-	5	0.51	1.23	5.39		3	2	2	15.5	8.84	
PF AA	6	0.6	1.18	6.67		3	2	2	18	6.7	98
PF AB	7	0.64	1.15	7.5		3	3	3			

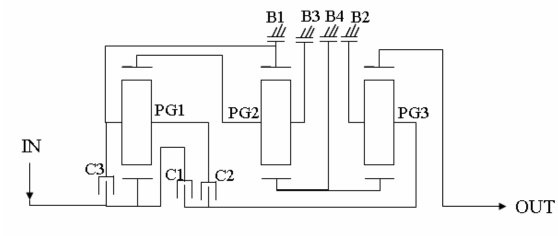
Figure 1: Traffic Light Chart

From the above chart it can be seen that PF A has the most desirable characteristics, but is not perfect. This approach can be applied to stepped ratio transmission, CVT's and hybrid powertrains

6 Results

Four representative designs have been selected for presentation here out of a candidate pool of over 100 designs. The details of these 4 designs are as follows.

1. Multi-Speed Concept 1 (Fig. (2)): This transmission (see [16]) is an 8-speed design that uses 3 simple planetary gear sets, 3 rotating clutches and 4 grounding clutches. It has 3 overdrives.



	Ratios	C1	C2	C3	B1	B2	B3	B4
Reverse	-2.74			X		X		
Neutral	0.00					X		
1	5.26					X	X	
2	2.67				X	X		
3	1.88		X				X	
4	1.45		X					X
5	1.00	X	X					
6	0.73	X						X
7	0.64	X					X	
8	0.58	X			X			

(X = engaged clutch)

Sample Design: $\frac{N_{R_1}}{N_{S_1}} = 3.01, \frac{N_{R_2}}{N_{S_2}} = 2.09, \frac{N_{R_3}}{N_{S_3}} = 2.74,$

Ratio Spread	9.15
Ratio Steps	
Rev/1	-0.52
1/2	1.97
2/3	1.42
3/4	1.30
4/5	1.45
5/6	1.36
6/7	1.14
7/8	1.12

Figure 2: Multi-Speed Concept 1

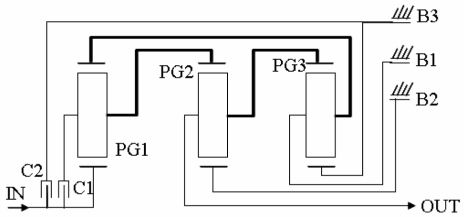
Features:

- All simple gear sets
- Only three rotating clutches, two of which are input clutches
- All clutches and brakes easily accessible to hydraulic circuitry
- Single transition clutching
- Direct drive

2. Multi-Speed Concept 2 (Fig. (3)): This transmission (see [17]), is a 6-speed design that uses 3 simple planetary gear sets, 2 rotating clutches and 3 grounding clutches. There is 1 overdrive in this design.

Features:

- All simple gear sets
- Two rotating input clutches
- Good ratio spread, torque ratios and steps
- Single transition clutching
- Direct drive



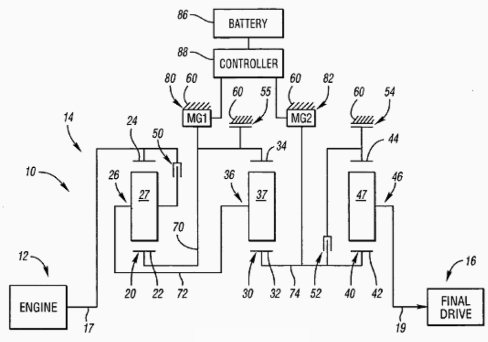
	Ratios	B1	B2	B3	C1	C2
Reverse	-2.93	X				X
Neutral	0.00	X				
1	4.66	X	X			
2	2.92		X	X		
3	1.83		X			X
4	1.40		X		X	
5	1.00				X	X
6	0.75			X	X	

(X = engaged clutch)

Sample Design: $\frac{N_{R_1}}{N_{S_1}} = 2.33, \frac{N_{R_2}}{N_{S_2}} = 2.52, \frac{N_{R_3}}{N_{S_3}} = 2.93$

Ratio Spread	6.25
Ratio Steps	
Rev/1	-0.63
1/2	1.60
2/3	1.59
3/4	1.31
4/5	1.40
5/6	1.34

Figure 3: Multi-Speed Concept 2



	RATIOS	50	52	80	82	54
Batt. Rev.	-3.25			0.00	-1.00(M)	X
EVT Rev.	-8.33			-0.39	-3.17(M)	X
TC Rev.	-7.00			-0.51	-2.84(M)	X
TC For.	4.69			-0.39	0.83(M)	X
Range 1.1	4.69			-0.39	0.83(M)	X
Range 1.2	3.45			-0.39	0.45(M)	X
Range 1.3	2.53			-0.39	0.17(M)	X
Range 1.4	1.86			-0.39(M)	-0.04	X
Range 2.1	1.36		X	-0.02	0.04(M)	
Range 2.2	1.00		X	0.04(M)	-0.04	
Range 2.3	0.74		X	0.16(M)	-0.051	
Range 2.4	0.54		X	0.48(M)	0.09	
F1	3.26	X				X
F2	2.00			X		X
F3	1.00	X	X			
F4	0.61		X	X		

Dual Mode (HS) (X = ENGAGED CLUTCH)

SAMPLE DESIGN:

$\frac{\text{RING GEAR}}{\text{SUN GEAR}}$ TOOTH RATIO:

$\frac{N_{R1}}{N_{S1}} = 1.63,$

$\frac{N_{R2}}{N_{S2}} = 1.63,$

$\frac{N_{R3}}{N_{S3}} = 2.26$

FIXED-RATIO MODE	
RATIO SPREAD	5.34
RATIO STEPS	
1/2	1.63
2/3	2.00
3/4	1.64

Figure 4: EVT Concept 1

3. EVT Concept 1 (Fig. (4))

This transmission (see [18]) is a full-function EVT comprised of 3 simple planetary gear sets, 2 rotating clutches, 2 stationary clutches, and 2 motor-generators, labeled MG1 and MG2. It operates in Battery Reverse, EVT Reverse and Forward, Battery-charging Reverse and Forward, and has 4 fixed (i.e., all mechanical) speed-ratios.

Features:

- All simple gear sets
- Acceptable pinion speeds
- Low electrical power losses
- Dual mode operation

4. EVT Concept 2 (Fig. (5))

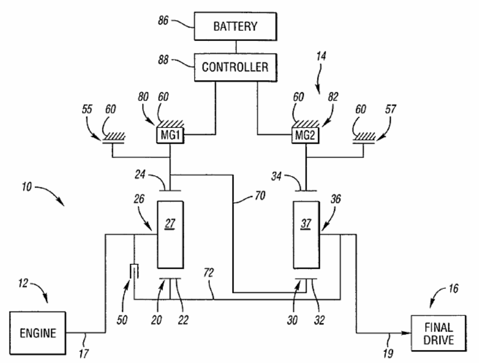
This transmission (see [19]) is a full-function EVT that uses 2 simple planetary gear sets, 1 rotating clutch, 2 stationary clutches, and 2 motor-generators, labeled MG1 and MG2. It operates in Battery Reverse, EVT Reverse and Forward, Battery-Charging Reverse and Forward, and has 3 fixed (i.e., all mechanical) speed-ratios.

Features:

- All simple gear sets
- All clutches easily accessible to hydraulic circuitry
- Acceptable pinion speeds
- Acceptable electrical power losses
- Single mode operation

7 Summary

We have used an algebraic synthesis procedure to generate several multi-speed transmission and EVT candidate designs. Sample concepts from the set generated are shown in this paper. The procedure allows the designer to generate and assess novel designs. It often proposes unusual arrangements, which even experienced designers might overlook. The process makes use of algebraic representation of transmission gear trains, graph-based searching and sorting, and transmission powerflow analyses. The computer-based procedure complements the traditional bag of tricks of the experienced transmission designer. Furthermore, as the requirements on fuel economy and performance compel manufacturers to use transmissions with higher numbers of speed ratios, it allows designers have to tackle increasingly complex mechanisms. Another benefit is its ability to identify minimum- content designs, wherein the emphasis is on achieving the maximum level of functionality with the fewest components.



	RATIOS	50	80/55	82/57
Batt. Rev.	-2.84		X	-1.00(M)
EVT Rev.	-8.33		-3.70	-5.63(M)
TC Rev.	-7.00		-3.24	-4.76(M)
TC For.	4.69		0.88(M)	2.81
Range 1.1	4.69		0.88(M)	2.81
Range 1.2	3.45		0.44(M)	1.99
Range 1.3	2.53		0.12	1.41(M)
Range 1.4	1.86		-0.12	0.98(M)
Range 1.5	1.36		-0.29	0.66(M)
Range 1.6	1.00		-0.43	0.42(M)
Range 1.7	0.74		-0.51	0.25(M)
Range 1.8	0.54		-0.58	0.12(M)
F1	2.19			X
F2	1.00	X		
F3	0.35		X	

SAMPLE DESIGN:

$\frac{\text{RING GEAR}}{\text{SUN GEAR}}$ TOOTH RATIO:

$\frac{N_{R1}}{N_{S1}} = 1.84$

$\frac{N_{R2}}{N_{S2}} = 1.84$

FIXED-RATIO MODE	
RATIO SPREAD	6.26
RATIO STEPS	
1/2	2.19
2/3	2.86

Figure 5: EVT Concept 2

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