Shaft Crack Identification using Artificial Neural Networks and Wavelet Transform data of a Transient Rotor

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Abstract

The dynamics and diagnostics of cracked rotors have been gaining importance in recent years. The present work deals with the detection of crack in rotor system and determination of crack parameters i.e. position of crack, depth of crack using Artificial Neural Networks (ANN). The detection of crack is based on the continuous wavelet transform (CWT) plots with 1/3 and ½ critical peaks as the crack indicators. The input to the CWT is the transient data which is obtained from the cracked run-up rotor with stepped acceleration, which is encountered in real time situations during the operation of steam turbines, aero-engines etc. The amplitudes of sub-critical and critical peaks of the CWT plots are fed as input to the neural network to obtain the crack parameters. The new concept in the present work is that both qualitative and quantitative treatment has been applied to diagnose the system and by using a simple data modification technique the number of sensors have been deduced from 21 to 3 for a 20 element FEM rotor model.

Keywords: Cracked Rotor, ANN, Wavelet Transform, Diagnostics.

1. Introduction

With the increasing need for high speed rotating machinery there has been an ever increasing demand for efficient diagnostic tools, so as to optimize the rotor system performance. Of all the usual faults like unbalance, misalignment etc. encountered in the rotor systems, the fatigue crack is the most dangerous one because if left undetected it can lead to catastrophic failure.

Vibration monitoring has proved to be a powerful diagnostic tool as it facilitates online detection of faults in the system. A vast amount of research has been carried out on the dynamic behavior of rotor-bearing systems with cracked shafts and excellent review on the same are available in [1, 2, 3]. Much of the previous work focused on the analysis of the steady state response for the crack detection. But it has been found that the start up and shut down signals of a rotor which contain the transient vibration information of the rotor can also contribute for detecting the crack as explained in [4].

The present study is an extension of the work done in [4] where the cracked rotor system was passed through its critical speed with a single acceleration which is a theoretical case. Here we are focusing on the transient response of a cracked rotor with stepped acceleration which is encountered in systems like aero-engines, steam turbines etc. Continuous Wavelet transform has been utilized to extract the information regarding crack.

The determination of crack parameters plays an important role in diagnostics of rotor systems. Genetic algorithms are used in [5] to solve the inverse problem of crack identification; the information about crack is obtained from the difference in natural frequencies of the cracked and uncracked beam. But the change in natural frequencies due to small crack depths is almost negligible, so the detection of small cracks is very difficult.

The diagnosis of the systems is carried in two ways, the qualitative treatment and quantitative treatment. But in the present work both the treatments are applied, so the detection as well as the determination of the parameters has been carried out in a single method. The CWT plots with 1/3 and ½ critical peaks is a qualitative indication of the crack in the system [6]. The amplitudes of the sub-critical peaks are fed as inputs to the neural network which is the quantitative treatment.

2. System Equation of Motion

The finite element model of a simple rotor-bearing system has been considered in [4] as shown in Fig. (1). The details of the cracked element used in the rotor shaft are given in Fig. (2). Here 'L' is the total length of the shaft, 'D' the diameter, 'l' the element length, 'm' mass of the disc, 'z' location of the crack and 'α' the crack depth.

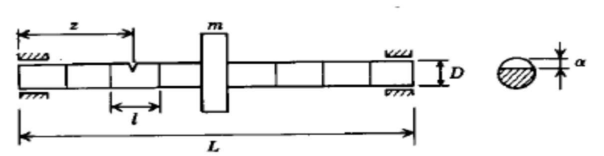

Figure 1: A finite element model of the rotor

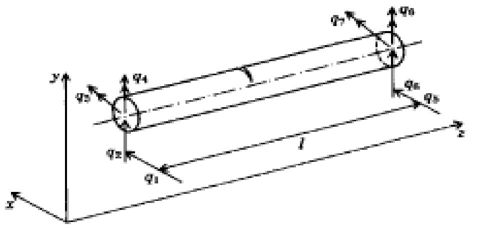

Figure 2: A cracked element

The equation of motion of the complete rotor system in a fixed coordinate system can be written as

$$[M]\{\ddot{q}\}+[D]\{\dot{q}\}+[K]\{q\}=\{F\} \qquad (1)$$

The excitation matrix $\{F\}$ consists of the unbalance forces due to disc having mass m, eccentricity e and the weight of the disc.

3. Crack Modeling

The transverse breathing crack [4] has been considered in the present study. With the shearing action neglected, and by using the strain energy, the flexibility coefficients for a bending element without a crack can be derived as in C_0 given below,

$$C_0=\begin{bmatrix} l^3/3EI & 0 & 0 & l^2/2EI \\ 0 & l^3/3EI & l^2/2EI & 0 \\ 0 & l^2/2EI & l/EI & 0 \\ l^2/2EI & 0 & 0 & l/EI \end{bmatrix}$$

Here EI is the bending stiffness. The breathing action of the crack [4] i.e. it's opening and closing is illustrated in Fig. (3). During the shaft's rotation, the crack opens and closes, depending on the rotor deflection. For a large class of machines, the static deflection is much greater than the rotor vibration.
With this assumption, the crack is closed when $\Phi = 0$ and it is fully open when $\Phi = 180$. The transverse surface crack on the shaft element introduces considerable local flexibility due to strain energy concentration in the vicinity of the crack tip under load. The additional strain energy due to the crack results in a local flexibility matrix which will be C_c, C_{op} and C_{HC} for a fully open crack and half-open, half-closed conditions respectively:

$$C_{op}=\left(\frac{1}{F_0}\right)\begin{bmatrix} \overline{C}_{11}R & 0 & 0 & 0 \\ 0 & \overline{C}_{22}R & 0 & 0 \\ 0 & 0 & \overline{C}_{33}/R & \overline{C}_{43}/R \\ 0 & 0 & \overline{C}_{43}/R & \overline{C}_{44}/R \end{bmatrix}$$

$$C_{HC}=\left(\frac{1}{2F_0}\right)\begin{bmatrix} \overline{C}_{22}R & 0 & 0 & 0 \\ 0 & \overline{C}_{11}R & 0 & 0 \\ 0 & 0 & \overline{C}_{44}/R & \overline{C}_{34}/R \\ 0 & 0 & \overline{C}_{34}/R & \overline{C}_{33}/R \end{bmatrix}$$

Here $F_0 = \pi E R^2 / (1-\upsilon^2)$, $R = D/2$ and υ = 0.3. The dimensionless compliance coefficients $\overline{C}_{ij}$ are functions of non-dimensional crack depth $\overline{\alpha}(where\,\overline{\alpha} = \alpha / D)$. This compliance coefficients are computed from the derivations discussed in [4]. The total flexibility matrix for the cracked section is given as

$$[C]=[C_0]+[C_c] \qquad (2)$$

The stiffness matrix of the cracked element [4] is written as

$$[K_c]=[T][C]^{-1}[T]^T \qquad (3)$$

where $[T]$ is the transformation matrix obtained from the equilibrium condition.

$$T=\begin{bmatrix} -1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & l & -1 & 0 \\ -l & 0 & 0 & -1 \\ 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

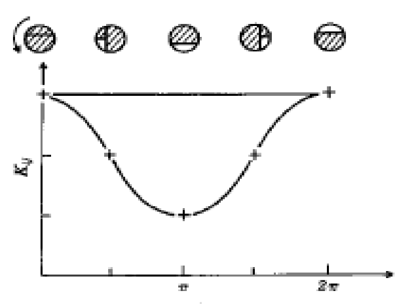

Figure 3: Breathing crack model

$$(q_1, q_2, \ldots\ldots, q_8)^T = [T](q_5, \ldots\ldots q_8)^T \qquad (4)$$

When the shaft is cracked, during rotation the stiffness varies with time, or with angle.

The variation may be expressed by a truncated cosine series

$$[K]=[K_0]+[K_1]\cos\omega t+[K_2]\cos 2\omega t+[K_3]\cos 3\omega t + [K_4]\cos 4\omega t \quad (5)$$

where, $[K_\eta]$, $\eta = 0, 1, \ldots, 4$ are the fitting coefficient matrices, determined from the known behavior of the stiffness matrix at certain angular locations as explained in [4].

4. Details of ANN

Artificial neural networks provide a general, non-linear parameterized mapping between a set of inputs and a set of outputs. A network with three layers of weights and sigmoidal activation functions can approximate any smooth mapping and such a type has been used here. A typical supervised feed-forward multi-layer neural network, known as a BP (Back Propagation) neural network has been used in the present analysis [7].

The network consists of three types of layers: the input layer (3 neurons – 3 inputs) that receives the sub-critical and critical data, the hidden layer (7 neurons – trial and error) which processes the data and the output layer (1 neuron – 1 output) that provides the result of the analysis, i.e., depth of crack. The training of a BP neural network is a two-step procedure. In the first stage, the network propagates input through each layer until an output is generated. The error between the actual output and the target output is then computed using mean square error (MSE). In the second stage, the calculated error is transmitted backwards from the output layer and the weights are adjusted accordingly in order to minimize the error. The training is stopped when the assumed value of the mean square error (MSE) is reached [8]. The structural damage detection model is established through network training on known samples by the Levenberg-Marquardt learning algorithm [8]. The performance of a trained network can be measured to some extent by the errors on the training, validation, and test sets, but it is often useful to investigate the network response in more detail. One option is to perform a regression analysis between the network response and the corresponding targets.

The network output and the corresponding targets are passed to regression analysis. It returns three parameters. The first two correspond to the slope and the y-intercept of the best linear regression relating targets to network outputs. If there were a perfect fit (outputs exactly equal to targets), the slope would be 1, and the y-intercept would be 0. The third variable is the correlation coefficient (R-value) between the outputs and targets. It is a measure of how well the variation in the output is explained by the targets. If this number is equal to 1, then there is perfect correlation between targets and outputs.

5. Results and Discussions

A steel shaft supported on two isotropic flexible bearings at both ends and having a disc at the centre, the following data is considered for the analysis: shaft diameter 20mm, length 780 mm; disc mass 5.5 kg, polar moment of inertia = 0.01546 kgm^2, unbalance eccentricity = 0.01 mm; bearing stiffness = 10^5 N/m, damping = 100 Ns/m.

The acceleration scheme followed is 30, 0, 200 rad/s^2 for 3, 2, 1.125 seconds respectively. With these accelerations the rotor system passes its first critical speed (178 rad/s) and reaches its operating speed of 315 rad/s. The Houbolt time marching technique with a time step of Δt = 0.001 s has been used to model the system in the time domain. The Morlet mother wavelet has been chosen for all the CWT's, with a scale of 40.

5.1 Crack Detection using CWT

The sample result plots for centre crack are shown below. In most of the cases the complete information of the signal is not observed in the time signal alone. So, it has to be processed or transformed into a different domain to observe the hidden features in the time signal. Here the crack in the system introduces some frequency components (1/3 and ½ critical peaks), which are clearly observed in a CWT signal [6], which is time-Frequency signal. The Figs. (4 (i)) & (4 (ii)) are the time and CWT plots for the system with no crack respectively. It is observed that the CWT plot (Fig. 4(ii)) shows no sub-harmonics for an un-cracked case. Figs. (5 (i)) & (5 (ii)) shows the time and CWT plots for a crack depth of 0.15 (crack depth/shaft dia.); the sub-criticals in the CWT is a clear indication of crack in the system [6]. The appearance of these sub-harmonics in the CWT plots is due to the breathing effect of crack in the system. The breathing action of the crack continuously changes the stiffness of the cracked element means the stiffness of the whole system. This change in the stiffness may be the reason for the appearance of sub-critical components, because the dynamic behavior of the system depends on the stiffness characteristics of the system. The same has been seen in Figs. (6 (i)) & (6 (ii)) but with a crack depth of 0.3. As the crack depth increases there is one more feature that can be observed from the CWT plot, which is the waviness of the envelope. This waviness can be attributed to the stepped acceleration effect and nothing else. Since these are not observed in the same cracked system with single acceleration. Because of the effect of stepped acceleration, the increase in crack depth not only enhances the maximum amplitudes of the sub-criticals but also introduces waviness to the envelope as shown in Figs. (5 (ii)) & (6 (ii)).

5.2 A Technique for Sensor Optimization and ANN

The amplitudes of these sub-criticals and criticals peaks are passed as an input to the ANN. To determine both the crack position and depth, the data have to be collected from each element (sensors) [7] along the rotor shaft. The implementation of this method in the real time rotor systems is practically difficult and also costly, because large number of sensors is required to collect the data. So to solve the above difficulty a simple data modification technique has been applied. With this technique the number of sensors has been drastically reduced from 21 to 3 per 0.5 meter shaft.

The technique is as follows, the sensors have been arranged in three different positions at 1/3 (left), ½ (centre), and 2/3 (right) lengths along the rotor shaft. The data collected from these three positions are used to find the position of the crack in the system. If the crack is at the centre of the system, the rotor deflection shape is symmetric. If the crack position shifts from the centre the symmetric shape tends to become asymmetric. This asymmetry in shape is calculated using the data collected from the above three sensors, based on the amplitude (CWT coefficients) and difference between the amplitudes of the sensors at 1/3 and 2/3 positions.

If the difference between the amplitudes is close to zero then the crack can be considered as near to the centre. The sample results have been shown in Table 1. The increase in the difference between the maximum amplitudes of the sub-criticals of the left and right sensor along the shaft shifts the crack from the centre. The element to which it is shifted depends on the magnitude of the difference. The advantage of this method is that it depends on the difference of the amplitudes which is a qualitative treatment. If the difference of the amplitudes results in two options, like crack in 10th element with 0.15 depth or in 8th element with 0.10 depth as shown in Table 1, then the absolute amplitude of the sub-critical collected from the middle sensor is compared with the amplitudes of the above two options. This results in finding the correct position of the crack. The error limit of the present technique in finding the position of the crack is below 5%.

After finding the position of the cracked element, the simulated data for the above cracked element is passed to the ANN to train the Network. Once the Network has been properly trained the test data is passed to obtain the depth of the crack. The Fig. (7) is the regression plot obtained for the test data of the system with crack at the centre. The X-axis shows the target and the y- axis shows the output of the ANN. The three variables of the regression analysis for the present case is slope is 0.97, Y- intercept is 0.42 and the correlation coefficient (R- value) is 0.99927. The R-value clearly says that the present fit is a very good fit.

Table 1: A sample wavelet data in the modified form for sensor optimization (10th element is the element near the centre for a 20 element rotor system).

A	B	C	D	E
5	9.759	28.83	0.233	1.11
6	13.79	41.07	0.21	1.16
7	18.48	55.39	0.14	0.98
8	23.95	71.57	0.12	0.67
10	36.47	109.7	0.02	0.13

Where,

A: Element number
B: Absolute amp. of 1/3 sub-critical of centre sensor with 0.10 crack depth
C: Absolute amp. of 1/3 sub-critical of centre sensor with 0.15 crack
D: Diff. in the amp. of 1/3 sub-critical of left and right sensor with 0.10 crack depth
E: Diff. in the amp. of 1/3 sub-critical of left and right sensor with 0.15 crack depth

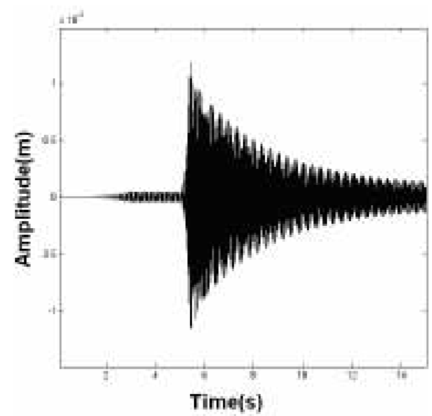

Figure 4 (i): Time plot for no crack

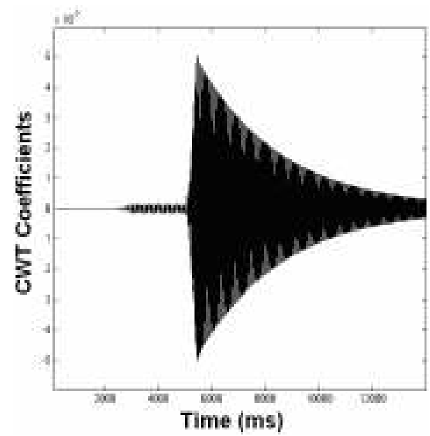

Figure 4 (ii): CWT plot for no crack

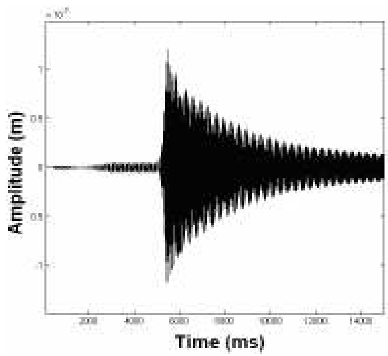

Figure 5 (i): Time plot for 0.15 crack

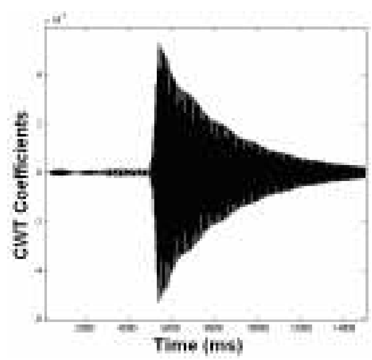

Figure 5 (ii): CWT plot for 0.15 crack

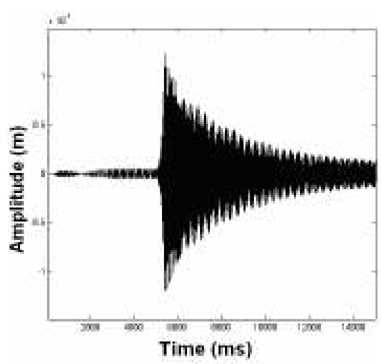

Figure 6 (i): Time plot for 0.3 crack

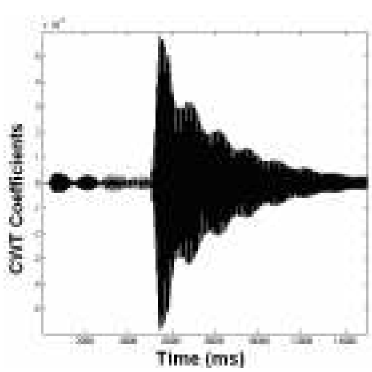

Figure 6 (ii): CWT plot for 0.3 crack

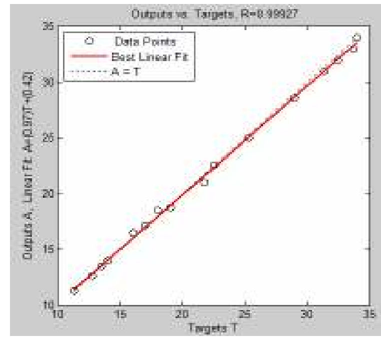

Figure 7: Regression plot for centre crack data

6. Conclusions

The transient analysis of rotor system with transverse breathing crack has been studied for flexural vibrations. The stepped acceleration applied to the system represents the real time system. The data modification technique found in the present work drastically reduces the requirement of number of sensors. The crack parameters have been identified using ANN and CWT data.

7. References

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