13th National Conference on Mechanisms and Machines (NaCoMM07),
IISc, Bangalore, India, December 12-13, 2007

NaCoMM-2007-103

Type Synthesis of Complaint 5-bar Mechanisms With Application to Mechanical Disc Brakes

Scott H. Brooks, Spencer P. Magleby*, Peter Halverson, and Larry L. Howell
Mechanical Engineering Department, Brigham Young University, Utah, USA

* Corresponding author (magleby@byu.edu)

Abstract

This paper presents the results of applying type synthesis to generate the possible configurations of a compliant 5-bar mechanism. The synthesis approach is described and demonstrated. Application of the approach resulted in the generation of 58 non-isomorphic configurations that are described and illustrated. While the paper is motivated by the potential application of selected members of the configuration set in development of mechanisms for grasping with special focus on mechanical disc brake systems, the principles may be applied to provide a broad range of mechanisms for evaluation and for intellectual property protection .

Keywords: Compliant mechanisms, Type synthesis

1 Introduction

Brooks et al. [1] presented grasping mechanism configurations that are self-centering (the grasping links center about the object being grasped) and force-balancing (reaction forces on an object being grasped or clamped are equal and opposite), and developed design principles and methods of achieving these characteristics in compliant mechanisms. The research was prompted by a desire to emulate, in a purely mechanical system, the self-centering and force-balancing characteristics exhibited by hydraulic disc-brake systems.

Clamp or grasp mechanisms are an interesting subset of self-centering mechanisms because they are tolerant of positional variation in the initial location of the workpiece. In other words, it does not matter if the workpiece (the object being grasped) is not already at the theoretical midpoint between the grasping (or output) links of the clamp. The workpiece will move to balance forces in the system. Assuming that the workpiece is oriented correctly, that it lies initially within the range of the grasping links and that only relative translations between the workpiece and the mechanism in the x-direction are allowed, the workpiece will center between the grasping links (or, the grasping links will center about the workpiece).

Attractive configurations of these grasp mechanisms incorporate compliant mechanisms with pseudo-rigid-body models that are 5-bar mechanisms (hereafter referred to as “compliant 5-bar mechanisms”). The objective of this paper is to present a type synthesis of compliant 5-bar mechanisms that is useful to designers working with grasping-type mechanisms.

2 Background

To better understand this research we provide a review of information related to: (1) Compliant mechanisms, (2) Self-centering and Force-balancing mechanisms, and (3) Methods of compliant mechanism synthesis.

2.1 Compliant Mechanisms

Compliant mechanisms are mechanisms that gain some or all of their motion from the deflection of flexible members [2]. They can often perform the same functions as rigid body mechanisms, but usually with many fewer parts and reduced assembly. Compliant mechanisms tend to reduce part complexity and weight through elimination of revolute joints and springs, which are common in most mechanisms. They also store most of the energy used during deflection and often result in greater precision through elimination of friction and wear surfaces common in revolute joints.

A few compliant grasping or workpiece holding devices have been developed that exhibit self-centering characteristics and are force balancing. It is uncommon to find an example of a self-centering mechanism that incorporates compliant mechanism technology to achieve functionality. One mechanism that does so is the Tektro MT20 compliant mountain bike brakes, shown in Fig. 1. The brakes were developed and analyzed by Mattson et al. [3] using the pseudo-rigid body model (PRBM). They are self-centering, and they incorporate a compliant steel spring to create a compliant mechanism that keeps the pad horizontal during brake actuation.

2.2 Self-Centering and Force-Balancing Mechanisms

The concept mechanism configuration in Fig. 2 was developed in an effort to produce a purely mechanical mechanism that approximates the centering and force-balancing characteristics of the hydraulic disc-brake. Looking at the floating-caliper disc-brake configuration in Fig. 2a, the brake may be modeled kinematically as in Fig 2b. A toggle linkage connects the piston (slider) to

Figure 1: Tektro MT20 Brakes incorporating a compliant link to create a parallel mechanism. Left photo source: http://www. tektro.com/mtb/mt20.htm.

the caliper (box slider), which travels in the positive x direction to contact the left side of the workpiece. Kinematically, this is the same as having two workpieces, or mechanical stops, and modeling the configuration in Fig. 2b as a double slider linkage, as shown in Fig. 2c.

The double slider linkage in Fig. 2c balances reactions, because the sliders are capable of transmitting forces only in the x direction. However, sliders are often undesirable for their manufacturing and operational limitations (tolerance and lubrication requirements, wear, etc.) The sliders may be further modeled with links of finite length to approximate the motion of the sliders for small angles. In this case it is acceptable because the piston and caliper displace only very small linear distances. The sliders of Fig. 2c may be approximated with links of finite length (a parallel-guiding four-bar mechanism replaces slider B, and a single link replaces slider A), to arrive at the seven-bar mechanism shown in Fig. 3.

2.3 Compliant Mechanism Synthesis

A review of compliant mechanism synthesis will be instructive during generation of concept mechanisms exhibiting the centering and force-balancing characteristics. Many papers, theses, and dissertations have been published on compliant mechanism synthesis including rigid-body replacement, type synthesis, and topological synthesis. Howell and Midha [4] present a generalized loop closure theory for the analysis and synthesis of compliant mechanisms. Murphy, Midha, and Howell [5] discuss type-synthesis of compliant mechanisms through simplified approaches to segment types. The authors present a simplified type-synthesis methodology that limits the number of design solutions to a given problem. The techniques are derived by modifying existing compliant type-synthesis techniques to yield a simpler and more pragmatic model.

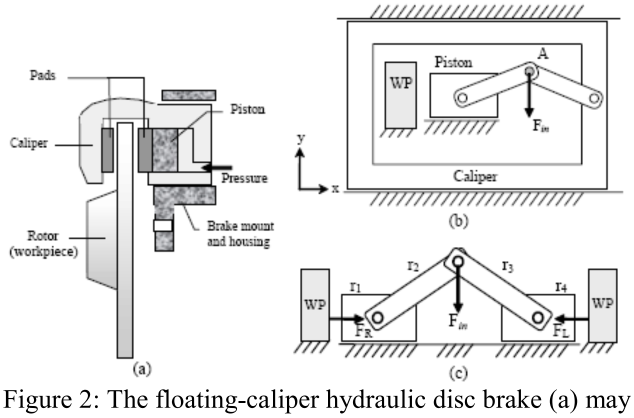

Figure 2: The floating-caliper hydraulic disc brake (a) may be modeled kinematically as in (b), which is a double slider linkage (c). A simple toggle linkage in combination with F_{in} has replaced the input pressure.

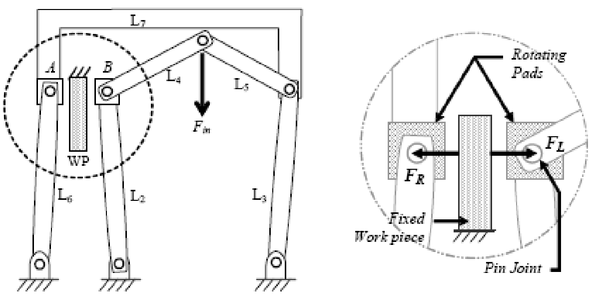

Figure 3: Seven bar, 2 degree of freedom mechanism. Links 6, 7, and 3 form a parallel 4-bar mechanism, while links 2, 4, 5 and 3 form a 5-bar. The workpiece is grasped between pin joints *A* and *B,* and exerts reaction forces F_L and F_R.

Murphy et al. [6] presents a systematic mathematical procedure for the topological synthesis of compliant mechanisms. According to Murphy, a topological synthesis is the first phase in the type synthesis of compliant mechanisms. Topological synthesis is the enumeration of mechanism structures based on the nature of the motion, the rigid-body degrees of freedom, and the number of links.. Joo et al. [7] discuss topological synthesis using linear beam elements. Starting with general input/output and force/displacement requirements and constraints, an improved and robust objective function is presented along with its implementation into a network of linear beam elements. Advantages and disadvantages of the method are discussed as well. Ananthasuresh, Kota and Noboru also discuss synthesis strategies of compliant MEMS at length [8].

3 Compliant Five-bar Type Synthesis

Murphy [9] discusses at length type synthesis of compliant mechanisms and uses graph theory and kinematic graphs to form a matrix representation of the linkage. The number of links in the mechanism determines the size (order) of the matrix. The elements of the matrix are used to indicate the linkage (segment) type and/or the connection type between linkages, and to designate the ground link. For example, the diagonal elements of a matrix giving a segment-type indication uses the values given in Table 1. A matrix giving a connection-type indication would use the non-diagonal elements of the complaint mechanism. Table 2 lists the values used to convey the information regarding the type of connection between segments. For example, if segment *i* is connected to segment *j*, its element *c(i,j)* will be zero, one, two or three, depending on the connection type. The following briefly outlines the type synthesis process.

The first objective in type synthesis is the determination of design requirements. The requirements determine the extent of the enumeration process. Murphy lists several questions that, when answered, may help the designer establish the design goals and limit the enumeration process:

1. Is a particular segment required to be the ground segment?
2. Are any connections required to be kinematic pairs, flexural pivots, or clamped connections?
3. Are segments required to be rigid?
4. Are kinematic pairs allowed?
5. Are flexural pivots allowed as connections to compliant segments?

Once the requirements have been established, the topological synthesis of compliant mechanisms is accomplished through a four-step process. The first step is to enumerate the possible combinations of segment type (rigid or compliant) without regard to the ground segment or the type of connections between segments. After the possible segment combinations have been enumerated, the design requirements are investigated and isomorphic chains are removed from further consideration. The second step of the topological synthesis process is to enumerate all the possible combinations of connections between segments without regard to the segment types being connected. Although isomorphisms are not investigated after this phase of the design process, resulting compliant chains are investigated for conformance to requirements. For example, if a design requirement is that each mechanism must contain one kinematic pair, the enumerated chains are investigated to ensure that at least one kinematic pair is present. The third step of the topological synthesis process is to combine the results of the segment and connection-type enumeration processes. The subsequent kinematic chains are grouped by the original compliant chain from the segment enumeration process. This grouping will help limit the extent of later isomorphism investigations. The connections between segments are now examined to remove any fixed connections between rigid segments and the chains are investigated to remove any isomorphisms. The fourth, and final, step of the topological synthesis process is to sequentially fix, or ground, each rigid segment to form mechanisms. If more than one mechanism is formed from a particular compliant chain, the mechanisms formed from that chain need to be investigated to ensure that they are unique (nonisomorphic). As with all the steps of the topological synthesis process, the applicable design requirements are enforced. The resulting mechanisms are forwarded for further investigation, which may include a topological analysis or ranking to determine which mechanisms will be selected for a particular application [9].

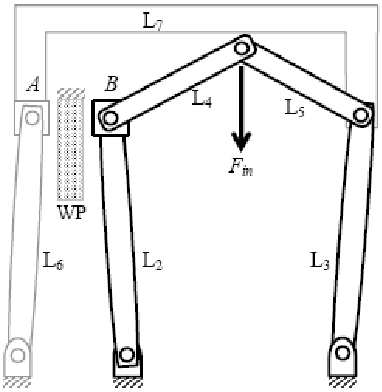

Figure 4: Five-bar portion of the 7-bar mechanism presented in Fig. 3.

Table 1: Segment-type Indication.

Segment Type	Diagonal element value, c(i,j)
Ground Segment	-1
Rigid Segment	0
Compliant Segment	1

Table 2: Connection-type indication (Murphy [9])

Connection Type	Matrix element Value, c(i,j)
None	0
Kinematic Pair	1
Flexural Pivot	2
Clamped Connection	3

This process was followed to arrive at the compliant mechanism configurations of Figs. 5 and 6. Fig. 5 shows 58 non-isomorphic compliant mechanism matrices for a 5-bar mechanism, resulting from compliant mechanism type synthesis. The 5 rows and columns of the matrices represent the 5 links in the mechanism. The matrix elements *Cij* are used to indicate the type of connection between links *i* and *j*. The ground link is designated with -1. A 0 means there is no connection; 1 designates a kinematic pair; 2 designates a flexural pivot; and 3 designates a clamped connection [9]. Fig. 6 graphically shows the corresponding compliant configurations represented by the matrices.

The type synthesis was based on the rigid-link 5-bar portion (links 1, 2, 4, 5 and 3) of the 7-bar mechanism shown in Fig. 4. The input must not be located on one of the independent links containing a generalized coordinate, because mobility of that degree of freedom (and the associated input force or displacement) will be lost upon contact with the workpiece. Thus, only links 4 and 5 may contain the input. The following lists all the topological design requirements, in the compliant mechanism type synthesis, resulting in Figs. 5 and 6.

1. Every configuration must contain 5 segments (links)

2. Links 4 and 5 must be rigid, because they experience high compressive loads and are assumed to be the input links
3. Flexural pivots are not allowed to be connected to flexible segments
4. The ground link is always rigid
5. A minimum of 2 compliant joints are required in each configuration

$$\underset{(a1)}{\begin{bmatrix}-1&2&0&0&1\\2&0&2&0&0\\0&2&0&1&0\\0&0&1&0&1\\1&0&0&1&0\end{bmatrix}}
\underset{(a2)}{\begin{bmatrix}-1&2&0&0&1\\2&0&1&0&0\\0&1&0&2&0\\0&0&2&0&1\\1&0&0&1&0\end{bmatrix}}
\underset{(a3)}{\begin{bmatrix}-1&2&0&0&1\\2&0&1&0&0\\0&1&0&1&0\\0&0&1&0&2\\1&0&0&2&0\end{bmatrix}}
\underset{(a4)}{\begin{bmatrix}-1&2&0&0&2\\2&0&1&0&0\\0&1&0&1&0\\0&0&1&0&1\\2&0&0&1&0\end{bmatrix}}
\underset{(a5)}{\begin{bmatrix}-1&1&0&0&1\\1&0&2&0&0\\0&2&0&2&0\\0&0&2&0&1\\1&0&0&1&0\end{bmatrix}}
\underset{(a6)}{\begin{bmatrix}-1&1&0&0&1\\1&0&2&0&0\\0&2&0&1&0\\0&0&1&0&2\\1&0&0&2&0\end{bmatrix}}$$

$$\underset{(b1)}{\begin{bmatrix}-1&2&0&0&1\\2&0&2&0&0\\0&2&0&2&0\\0&0&2&0&1\\1&0&0&1&0\end{bmatrix}}
\underset{(b2)}{\begin{bmatrix}-1&2&0&0&1\\2&0&1&0&0\\0&1&0&2&0\\0&0&2&0&2\\1&0&0&2&0\end{bmatrix}}
\underset{(b3)}{\begin{bmatrix}-1&2&0&0&2\\2&0&1&0&0\\0&1&0&1&0\\0&0&1&0&2\\2&0&0&2&0\end{bmatrix}}
\underset{(b4)}{\begin{bmatrix}-1&1&0&0&1\\1&0&2&0&0\\0&2&0&2&0\\0&0&2&0&2\\1&0&0&2&0\end{bmatrix}}
\underset{(b5)}{\begin{bmatrix}-1&2&0&0&2\\2&0&1&0&0\\0&1&0&2&0\\0&0&2&0&1\\2&0&0&1&0\end{bmatrix}}
\underset{(b6)}{\begin{bmatrix}-1&2&0&0&1\\2&0&2&0&0\\0&2&0&1&0\\0&0&1&0&2\\1&0&0&2&0\end{bmatrix}}$$

$$\underset{(c1)}{\begin{bmatrix}-1&2&0&0&1\\2&0&2&0&0\\0&2&0&2&0\\0&0&2&0&2\\1&0&0&2&0\end{bmatrix}}
\underset{(c2)}{\begin{bmatrix}-1&2&0&0&2\\2&0&2&0&0\\0&2&0&2&0\\0&0&2&0&1\\2&0&0&1&0\end{bmatrix}}
\underset{(c3)}{\begin{bmatrix}-1&2&0&0&2\\2&0&2&0&0\\0&2&0&1&0\\0&0&1&0&2\\2&0&0&2&0\end{bmatrix}}
\underset{(c4)}{\begin{bmatrix}-1&2&0&0&2\\2&0&2&0&0\\0&2&0&2&0\\0&0&2&0&2\\2&0&0&2&0\end{bmatrix}}$$

$$\underset{(d1)}{\begin{bmatrix}-1&2&0&0&1\\2&0&2&0&0\\0&2&0&1&0\\0&0&1&0&1\\1&0&0&1&1\end{bmatrix}}
\underset{(d2)}{\begin{bmatrix}-1&2&0&0&1\\2&0&1&0&0\\0&1&0&2&0\\0&0&2&0&1\\1&0&0&1&1\end{bmatrix}}
\underset{(d3)}{\begin{bmatrix}-1&1&0&0&1\\1&0&2&0&0\\0&2&0&2&0\\0&0&2&0&1\\1&0&0&1&1\end{bmatrix}}
\underset{(d4)}{\begin{bmatrix}-1&2&0&0&1\\2&0&2&0&0\\0&2&0&2&0\\0&0&2&0&1\\1&0&0&1&1\end{bmatrix}}$$

$$\underset{(e1)}{\begin{bmatrix}-1&3&0&0&1\\3&1&3&0&0\\0&3&0&1&0\\0&0&1&0&1\\1&0&0&1&0\end{bmatrix}}
\underset{(e2)}{\begin{bmatrix}-1&3&0&0&1\\3&1&3&0&0\\0&3&0&1&0\\0&0&1&0&1\\1&0&0&1&1\end{bmatrix}}
\underset{(e3)}{\begin{bmatrix}-1&3&0&0&1\\3&1&1&0&0\\0&1&0&1&0\\0&0&1&0&3\\1&0&0&3&1\end{bmatrix}}
\underset{(e4)}{\begin{bmatrix}-1&3&0&0&3\\3&1&1&0&0\\0&1&0&1&0\\0&0&1&0&1\\3&0&0&1&1\end{bmatrix}}
\underset{(e5)}{\begin{bmatrix}-1&1&0&0&1\\1&1&3&0&0\\0&3&0&1&0\\0&0&1&0&3\\1&0&0&3&1\end{bmatrix}}
\underset{(e6)}{\begin{bmatrix}-1&3&0&0&1\\3&1&3&0&0\\0&3&0&1&0\\0&0&1&0&3\\1&0&0&3&1\end{bmatrix}}$$

$$\underset{(e7)}{\begin{bmatrix}-1&3&0&0&3\\3&1&3&0&0\\0&3&0&1&0\\0&0&1&0&1\\3&0&0&1&1\end{bmatrix}}
\underset{(e8)}{\begin{bmatrix}-1&3&0&0&3\\3&1&3&0&0\\0&3&0&1&0\\0&0&1&0&3\\3&0&0&3&1\end{bmatrix}}$$

$$\underset{(f1)}{\begin{bmatrix}-1&2&0&0&3\\2&0&1&0&0\\0&1&0&1&0\\0&0&1&0&1\\3&0&0&1&1\end{bmatrix}}
\underset{(f2)}{\begin{bmatrix}-1&2&0&0&1\\2&0&1&0&0\\0&1&0&1&0\\0&0&1&0&3\\1&0&0&3&1\end{bmatrix}}
\underset{(f3)}{\begin{bmatrix}-1&1&0&0&3\\1&0&2&0&0\\0&2&0&1&0\\0&0&1&0&1\\3&0&0&1&1\end{bmatrix}}
\underset{(f4)}{\begin{bmatrix}-1&1&0&0&1\\1&0&2&0&0\\0&2&0&1&0\\0&0&1&0&3\\1&0&0&3&1\end{bmatrix}}
\underset{(f5)}{\begin{bmatrix}-1&1&0&0&1\\1&0&1&0&0\\0&1&0&2&0\\0&0&2&0&3\\1&0&0&3&1\end{bmatrix}}
\underset{(f6)}{\begin{bmatrix}-1&1&0&0&3\\1&0&1&0&0\\0&1&0&2&0\\0&0&2&0&1\\3&0&0&1&1\end{bmatrix}}$$

Figure 5: 58 non-isomorphic compliant mechanism matrices for a 5-bar self-centering, force balancing mechanism. Concepts with *(a)* 2 flexural pivots; *(b)* 3 flexural pivots; *(c)* 4 or 5 flexural pivots; *(d)* 2 or 3 flexural pivots and flexible links; *(e)* 2, 3 or 4 clamped connections; *(f)* 1 flexural pivot and 1 clamped connection.

$$\begin{bmatrix}-1&1&0&0&1\\1&1&1&0&0\\0&1&0&2&0\\0&0&2&0&3\\1&0&0&3&1\end{bmatrix}$$

(f7)

$$\begin{bmatrix}-1&1&0&0&3\\1&1&1&0&0\\0&1&0&2&0\\0&0&2&0&1\\3&0&0&1&1\end{bmatrix}$$

(f8)

$$\begin{bmatrix}-1&2&0&0&3\\2&0&2&0&0\\0&2&0&1&0\\0&0&1&0&1\\3&0&0&1&1\end{bmatrix}$$

(g1)

$$\begin{bmatrix}-1&2&0&0&1\\2&0&2&0&0\\0&2&0&1&0\\0&0&1&0&3\\1&0&0&3&1\end{bmatrix}$$

(g2)

$$\begin{bmatrix}-1&1&0&0&3\\1&0&2&0&0\\0&2&0&2&0\\0&0&2&0&1\\3&0&0&1&1\end{bmatrix}$$

(g3)

$$\begin{bmatrix}-1&1&0&0&1\\1&0&2&0&0\\0&2&0&2&0\\0&0&2&0&3\\1&0&0&3&1\end{bmatrix}$$

(g4)

$$\begin{bmatrix}-1&2&0&0&1\\2&0&1&0&0\\0&1&0&2&0\\0&0&2&0&3\\1&0&0&3&1\end{bmatrix}$$

(g5)

$$\begin{bmatrix}-1&2&0&0&3\\2&0&1&0&0\\0&1&0&2&0\\0&0&2&0&1\\3&0&0&1&1\end{bmatrix}$$

(g6)

$$\begin{bmatrix}-1&2&0&0&3\\2&0&1&0&0\\0&1&0&1&0\\0&0&1&0&3\\3&0&0&3&1\end{bmatrix}$$

(h1)

$$\begin{bmatrix}-1&1&0&0&3\\1&0&2&0&0\\0&2&0&1&0\\0&0&1&0&3\\3&0&0&3&1\end{bmatrix}$$

(h2)

$$\begin{bmatrix}-1&1&0&0&3\\1&0&1&0&0\\0&1&0&2&0\\0&0&2&0&3\\3&0&0&3&1\end{bmatrix}$$

(h3)

$$\begin{bmatrix}-1&1&0&0&1\\1&1&3&0&0\\0&3&0&2&0\\0&0&2&0&3\\1&0&0&3&1\end{bmatrix}$$

(h4)

$$\begin{bmatrix}-1&3&0&0&3\\3&1&1&0&0\\0&1&0&2&0\\0&0&2&0&1\\3&0&0&1&1\end{bmatrix}$$

(h5)

$$\begin{bmatrix}-1&1&0&0&3\\1&1&3&0&0\\0&3&0&2&0\\0&0&2&0&1\\3&0&0&1&1\end{bmatrix}$$

(h6)

$$\begin{bmatrix}-1&1&0&0&3\\1&1&1&0&0\\0&1&0&2&0\\0&0&2&0&3\\3&0&0&3&1\end{bmatrix}$$

(h7)

$$\begin{bmatrix}-1&3&0&0&3\\3&1&3&0&0\\0&3&0&2&0\\0&0&2&0&1\\3&0&0&1&1\end{bmatrix}$$

(i1)

$$\begin{bmatrix}-1&1&0&0&3\\1&1&3&0&0\\0&3&0&2&0\\0&0&2&0&3\\3&0&0&3&1\end{bmatrix}$$

(i2)

$$\begin{bmatrix}-1&3&0&0&3\\3&1&3&0&0\\0&3&0&2&0\\0&0&2&0&3\\3&0&0&3&1\end{bmatrix}$$

(i3)

$$\begin{bmatrix}-1&2&0&0&3\\2&0&2&0&0\\0&2&0&1&0\\0&0&1&0&3\\3&0&0&3&1\end{bmatrix}$$

(i4)

$$\begin{bmatrix}-1&1&0&0&3\\1&0&2&0&0\\0&2&0&2&0\\0&0&2&0&3\\3&0&0&3&1\end{bmatrix}$$

(i5)

$$\begin{bmatrix}-1&2&0&0&3\\2&0&1&0&0\\0&1&0&2&0\\0&0&2&0&3\\3&0&0&3&1\end{bmatrix}$$

(i6)

$$\begin{bmatrix}-1&2&0&0&3\\2&0&2&0&0\\0&2&0&2&0\\0&0&2&0&1\\3&0&0&1&1\end{bmatrix}$$

(j1)

$$\begin{bmatrix}-1&2&0&0&1\\2&0&2&0&0\\0&2&0&2&0\\0&0&2&0&3\\1&0&0&3&1\end{bmatrix}$$

(j2)

$$\begin{bmatrix}-1&2&0&0&3\\2&0&2&0&0\\0&2&0&2&0\\0&0&2&0&3\\3&0&0&3&1\end{bmatrix}$$

(j3)

Figure 5 Continued... Concepts with *(f)* 1 flexural pivot and 1 clamped connection; (g) 2 or 3 flexural pivots and 1 clamped connection; (h) 1 flexural pivot and 2 clamped connections; (i) 1 or 2 flexural pivot and 3 or 4 clamped connections; (j) 3 flexural pivots and 1 or 2 clamped connections.

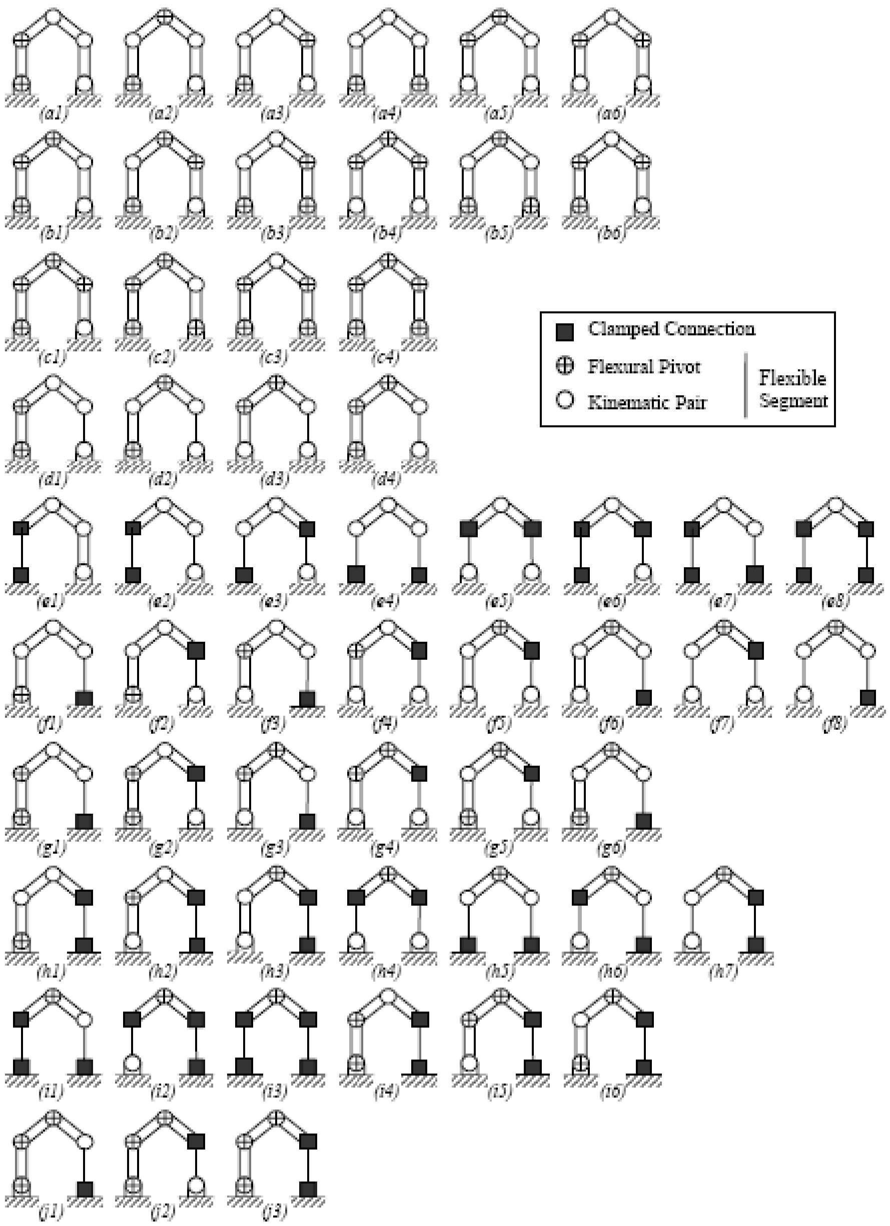

Figure 6: 58 configurations of the compliant 5-bar mechanism resulting from type synthesis. The corresponding matrices are shown in Fig. 5.

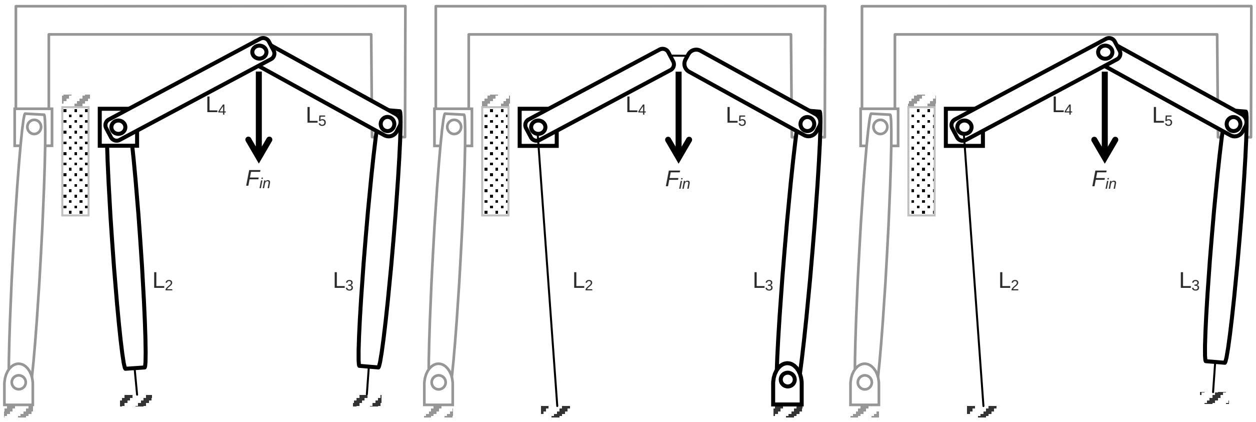

Figure 7: Three implementations (in the context of the mechanism in Figure 3) of the 58 possible configurations determined via type synthesis. These figures correspond (from left to right) with configurations a4, f5, and f1.

Embodiments of three configurations (a4, f5, f1), from the 58 possible configurations generated during type synthesis, are illustrated in Figure 7. These three embodiments help to show the broad range of mechanisms that could be used in self-centering and force-balancing compliant systems.

7 Conclusions

An important advantage gained by incorporating compliance into self-centering, force-balancing systems is that type synthesis techniques will yield sets or families of alternative compliant configurations which are unavailable if considering only rigid-link configurations. This research developed 58 non-isomorphic configurations for just the 5-bar mechanismType synthesis is useful because its use allows access to a wealth of previously unidentified, compliant mechanisms. The configurations presented here are now available for further development in later steps in the design process. One early effort in this area is described in [10].

While type synthesis does not evaluate designs, it does generate the complete set of possible design configurations. Embodiments from these configurations can be used to explore the design space, generate additional designs for intellectual property protection, or circumvent designs currently covered by intellectual property.

References

[1] Scott Brooks, Spencer Magleby and Larry Howell, *“Grasping Mechanisms with Self-Centering and Force-Balancing Characteristics,”* Proceedings of the 2005 ASME International Design Engineering Technical Conferences, Long Beach, CA, September 2005, DETC2005- 85478.

[2] Howell, L.L., *Compliant Mechanisms*, John Wiley & Sons, Inc., New York, 2001.

[3] Mattson C. A., Howell, L. L., Magleby, S. P., "Development of commercially Viable Compliant Mechanisms Using the Psuedo-Rigid-Body Model: Case Studies of Parallel Mechanisms." Proceedings of 2001 ASME International Mechanical Engineering Congress and Exposition, New York, NY, paper no. AD-23718.

[4] Howell, L.L.Midha, A., "Loop-Closure Theory for the Analysis and Synthesis of Compliant Mechanisms." *Journal of Mechanical Design*, Transactions of the ASME, Vol.118 no.1, 1996, pp.121-125.

[5] Murphy, Morgan D., Midha, Ashok and Howell, Larry L., "Type Synthesis of Compliant Mechanisms Employing a Simplified Approach to Segment Type." Proceedings of the 1994 ASME Design Technical Conferences. Part I of III. *American Society of Mechanical Engineers, Design Engineering Division*, Vol. 70 no.1, 1994, pp. 51-60.

[6] Murphy, Morgan D., Midha, Ashok and Howell, Larry L., "The Topological Synthesis of Compliant Mechanisms." *Journal of Mechanisms and Machine Theory*, Vol. 31, no.2, 1996, pp. 185-199.

[7] Joo, Jinyong; Kota, Sridhar; Kikuchi, Noboru. "Topological Synthesis of Compliant Mechanisms Using Linear Beam Elements." *Mechanics of Structures and Machines*, Vol. 28 no. 4, 2000, pp. 245-280.

[8] Ananthasuresh, G.K., Kota Sridhar, and Kikuchi, Noboru, "Strategies for Systematic Synthesis of Compliant MEMS." *American Society of Mechanical Engineers, Dynamic Systems and Control Division*, Vol. 55 no. 2, 1994, pp. 677-686.

[9] Murphy, Morgan D., Midha, Ashok and Howell, Larry L., "Methodology for the Design of Compliant Mechanisms Employing Type Synthesis Techniques With Example." *American Society of Mechanical Engineers, Design Engineering Division*, Vol. 70 no.1, 1994, pp. 61-66.

[10] Montierth, Jacob R., “Elliptical Rolling Link Toggle Mechanisms for Passive Force Closures with Self-Adjustment,” M.S. Thesis, Brigham Young University, 2007.