Error Analysis of Four Bar Planar Mechanism Considering Clearance Between Crank & Coupler

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Abstract

Joint clearance leads to mechanical error in mechanism. The problem has been addressed by a number of authors, who have analyzed clearance effect from different perspective. In this paper exact location of clearance link has been identified using dynamic analysis. Mechanical error in coupler point and output angle deviation due to clearance link and its exact location is dealt using sensitivity linkage and velocity analysis.

Keywords: Sensitivity linkage, Sensitivity co-efficient, Mechanical error and Clearance link.

1 Introduction

A mechanical system is made-up of several components, which can be divided into two major groups namely links that is bodies with convenient geometry and joints. Joints introduce some restrictions on the relative motion of the various bodies of the system. Joint clearances and link manufacturing tolerances produce mechanical error in linkages. To design a more accurate mechanism for a given task it is necessary to take into consideration the mechanical errors from initial design stages. And to do so, an efficient method is required to calculate the size of mechanical errors from a given mechanism.

The value of mechanical error may be higher than that of structural error and give rise to nonlinearity of relation between input and output displacements. Clearance does not have positive effects only : if on one hand it simplifies the assembling and the manufacturing of the mechanism, on the other hand it generates errors in the pose (position and orientation) of its links. Moreover, it is one of the main cause of shock between the mechanism links, and consequently on vibrations and noise. Evaluating the influence that clearance in the kinematics pairs has on mechanisms is an old problem, and many authors have proposed solutions based on different approaches. Denavit and Hartenberge analyzed mechanical error by deterministic method [1]; Kwun-Long, Jianmin Zhu & Devek Watkins presented an approach to identify the worst position & directions errors due to the joint clearances of linkages and manipulators [2] ; Jianmin Zhu & Kwun-Long Ting based on the probability theory, a general probability density function of the endpoint of planar robots is established by grouping the planar joint deviation vectors and establishing the structural constraint conditions between the vector groups [3]; Ming-June Tsai, Tien-Hising Lai dealt with an effective method to analyze the transmission performance of linkages that have joint clearance. Joint clearance was treated as virtual link to simplify the study [4]; Weidang Wu & S.S.Rao presented a new approach based on interval analysis for the modeling and analysis of tolerances and clearances in fuzzy error analysis of planar and spatial mechanisms [5] ; V. Parenti-Castelli & S. Venanzi presented method of kinematics modeling, assessing the actual configuration of a mechanism with clearance effected pairs [6] .

In the present work, error analysis of four bar planar mechanism for given clearance between crank & coupler is presented for mechanical error of the coupler point and error in output angle.

2 Mechanical Error And Sensitivity Co-efficient

Mechanical error of the path point P Fig. (1) resulting from a clearance at the pin joint B is defined as :

$$E_p(r, \delta) = R_p(r, \delta) - R_p(0, \delta) \quad (1)$$

r and δ represents the relative position of the pin within clearance circle Fig. (2). As the magnitude of the clearance is small compared to the lengths of links $R_p(r, \delta)$ can be approximated by Taylor's series

$$R_p(r, \delta)= R_p(0, \delta)+ \partial R_p/\partial r \,\big|_{(0,\, \delta)} * r \quad (2)$$

Eq. (1) & (2) gives

$$E_p(r, \delta) = \partial R_p/\partial r \,\big|_{(0,\, \delta)} * r \quad (3)$$

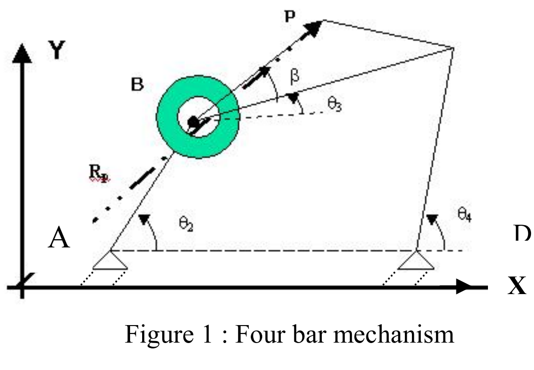

Figure 1 : Four bar mechanism

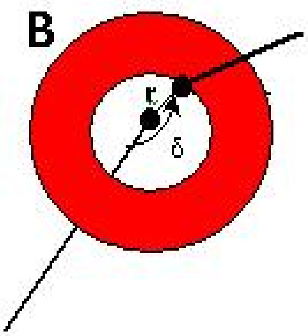

Figure 2 : position of the pin within clearance circle

The term $\partial R_p/\partial r$ is known as sensitivity co-efficient and represents the sensitivity of change of the path point P as the pin moves toward the direction represented by the angle δ.

3 Sensitivity Linkage

The new method which transforms the problem of calculating sensitivity coefficients into one of the velocity analysis of sensitivity linkage is dealt by Jae Kun Shin & Jin Han Jun [7]. In this, two more links are added to the original four bar mechanism Fig. (3) . A slider ' S' is pinned at B_2 and a link 'l_5' through the slider is pinned at B_1 to get a new linkage of 3-DOF. Let 'l_5' be fixed relative to the input link by an angle δ. Then sensitivity coefficients is given by

$$\partial R_p/\partial r\,|_{(0,\,\delta)} = \lim_{\Delta S \to 0} (\Delta R_p/\Delta S) = \lim_{\Delta S \to 0}(\Delta R_p/\Delta t/\Delta S/\Delta t) = V_p/s^1 \quad (4)$$

The condition for the velocity analysis is such that the input angle θ_2 and δ must be fixed and input velocity S^1 is given Fig. (4). As the velocity of certain point in mechanism is proportional to the input velocity. Required velocity ratio can be obtained by setting input velocity equal to one and Eq. (4) becomes

$$\partial R_p/\partial r\,|_{(0,\,\delta)} = V_p \mid \theta_2=\theta_2 ,\delta= \delta,S^1=S^1 \quad (5)$$

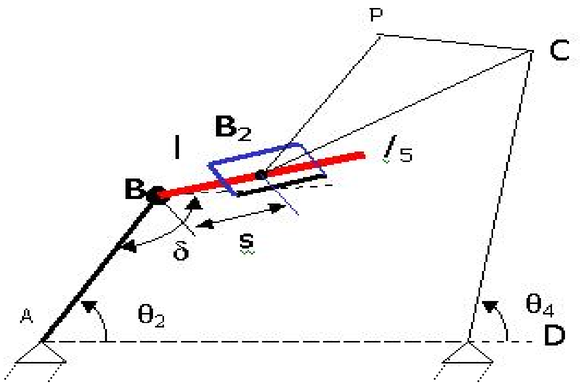

Figure 3: Sensitivity linkage for the clearance .

The problem of finding the velocity of the path P on the sensitivity linkage is equivalent to find the velocity P on this separated mechanism.
Velocity of point P is obtained as $V_P=V_B+V_{PB}$;

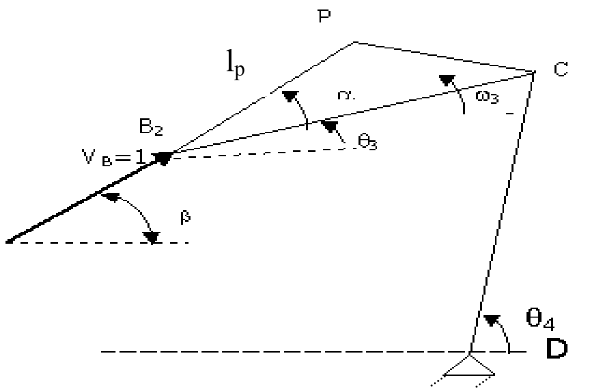

Figure 4: Open kinematics chain

$V_B=(\cos\beta , \sin\beta);V_{PB}=\{- lp\ \omega_3\sin (\theta_3+\alpha);\ lp\ \omega_3\cos (\theta_3+\alpha) \}$;

$$V_P = (\cos\beta-lp\ \omega_3\sin (\theta_3+\alpha); \sin \beta+lp\ \omega_3\cos (\theta_3+\alpha))^T \quad (6)$$

$$E_p(\delta) = V_p*r \quad (7)$$

The parameters defining the mechanism are listed in Table 1. Eq.(7) is solved for coupler point P of mechanism , for fixed crank angle $\theta_2 =30^0$ and complete one revolution of clearance link angle (δ). The mechanical error in X & Y component for coupler point P has been listed in Table 2 and depicted in Fig. (5).

Table 1: Details of four bar mechanism

	Crank	Coupler	Follower
Length (R_i); cm	10.80	27.94	27.05
Area (A_i);cm^2	1.077	0.406	0.406
Area Movement of inertia ;cm^4	$1.616*10^{-2}$	$8.674*10^{-4}$	$8.674*10^{-4}$
Distance between ground pivots;cm	25.4	———	———
Modulus of Elasticity; kPa	$7.10*10^7$	———	———
Clearance between Crank & Coupler; cm	0.105	———	———

Position of coupler Point (P) $\alpha=38^0$ w.r.t. coupler link and link (lp) is 5.0 cm. Weight density $2.66 *10^{-2}N/cm^3$. RPM of crank is 340.

Table 2: Mechanical Error for Crank Angles 30^0

Angle δ	Mechanical Error For 30^0	
	X-Comp.	Y-Comp.
0	-0.1035	-0.1035
30	-0.07734	-0.07734
60	-0.03044	-0.03044
90	0.024615	0.024615
120	0.073073	0.073073
150	0.101936	0.101936
180	0.103464	0.103464
210	0.077247	0.077247
240	0.030315	0.030315
270	-0.02475	-0.02475
300	-0.07317	-0.07317
330	-0.10197	-0.10197
360	-0.10343	-0.10343

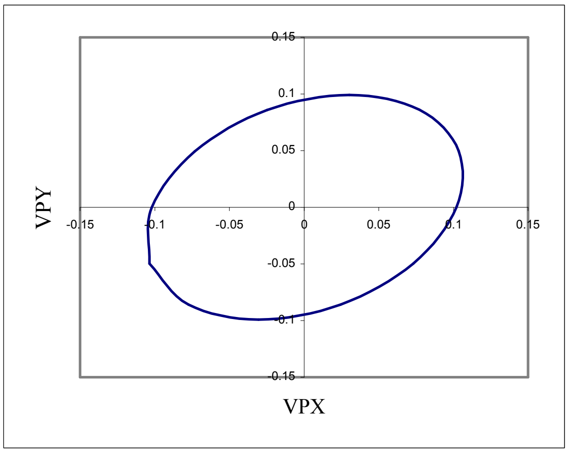

Figure 5: Distribution of Mechanical Error at $\Theta_2 = 30^0$ & δ varies 0-360^0 for coupler point P.

4 Clearances and Clearance Link Orientation

The method proposed by [7] gives distribution of mechanical error for a given crank angle and variation of clearance link angle from 0-360^0 Fig. (5). However, to locate the exact mechanical error for given crank angle it is necessary to find out the orientation of the clearance link (δ). Hence in the present paper an attempt is made to determine the clearance link position for finding out error vector by using an approach of Dynamic Analysis .

The position of the journal in the bearing is directly related to the amount of load on the bearing. When the journal bearing is sufficiently supplied with oil and with no load acting, the journal rotates concentrically with the bearing. However ,when the load is applied, the journal moves to equilibrium position and gives discontinuities in the motion resulting from clearances, eccentricities, aptitude angle etc . Complete motion of journal is described by sliding, free flight and impact mode between the tip of journal and bearing [11]. In the present analysis sliding mode is preferred, in which journal pin is in contact with bearing.

4.1 Clearance Link Orientation

The clearance can be allocated for acceptable deviation in output angle. By assuming small deviation in the link parameter to follows statistical pattern, the change in output angle is expressed as

$$\delta\theta_4 = \left\{ \sum_{i=1}^{n} \left[(\delta\theta_4/\delta v)_{i*}\, \delta v_i \right]^2 \right\}^{1/2} \tag{8}$$

Also assuming all tolerances and clearances have same effect upon deviation in output angle Eq. (8) can be written

$$\delta v_i = \delta\theta_4 / \sqrt{n}\, (\delta\theta_4/\delta v_i) \tag{9}$$

where allowable deviations in links parameters $\pm\delta v_i$ and $\pm\delta\theta_4$ is specified output deviation .

Once the clearance has been allocated or fixed to the joints of four bar mechanism, its orientation was adopted from joint force between the links of mechanism. The joint force is obtained from the analysis of kinematics and dynamics of four bar mechanism with the assumption of rigid –body motion of all linkages and zero clearance in all joints [8].

A standard approach to dynamics analysis of a linkage is the Matrix Method, developed by using D'Alembert principle [9,10]. In this method physical properties of ith link are specified by its mass M_i and the moment of inertia about the mass center I_i. The angular position velocity and acceleration analysis are obtained using LINKPAC [9] . The system of internal joint forces

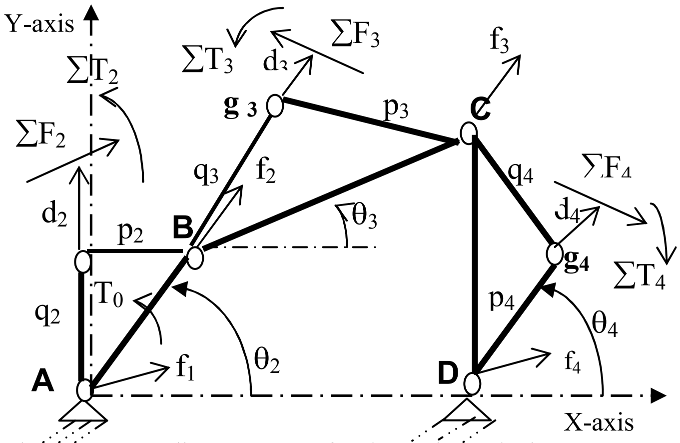

Figure 6: Coordinate System for dynamic analysis

f_i and required torque T_o are to be calculated by solving nine equations which are conveniently arranged in Matrix form. In Eq.(10) & Fig. (6), the sum of the external loads and torque acting on the ith link will be designed as $\sum F_i$ and $\sum T_i$. The resultant force acting on the ith link is the sum of the external loads $\sum F_i$ plus the two joint forces f_i (leading joint force) ,$f_{(i-1)}$ (trailing joint force). The equation of motion for the ith member can be written in the form as governed by Newton's second law;

$$f_i - f_{(i-1)} + \sum F_i = m_i \ddot{g}_i \tag{10}$$

$$(p_i * f_i) - (q_i\ f_{(i-1)}) + (d_i * + \sum F_i) + T_i = I_i * \ddot{\theta}_2 \tag{11}$$

where p_i is the vector from g_i to the leading joint i ;
q_i is the vector from g_i to the trailing joint i-1;
d_i is the vector from g_i to one point on $\sum F_i$.

Expression of Eq. (10) can be written in X & Y component of four bar linkage. The nine equation of motion are conveniently arranged in the matrix form. Square matrix on left side of Eq. (12) describes the instantaneous geometry of the mechanism. The column matrix on the right side contains the dynamic terms. The Matrix equation is solved by Gauss' elimination.

For given mechanism Table 1. the Eq. (12) is solved for joint force between crank and coupler to get the clearance link angle (δ) for one complete revolution of crank. For fixed crank angle 36^0 the joint force in X & Y direction are F_X = 8.64E+03 N ; F_Y =6.69E+03N. The clearance link angle (δ =37.75^0) can be obtained as

$$\delta = \tan^{-1}(F_X/F_Y) \tag{13}$$

$$\begin{bmatrix} -1 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & -1 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ q_{2y} & -q_{2x} & -p_{2y} & p_{2x} & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & -1 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & -1 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & -q_{3y} & -q_{3x} & -p_{3y} & p_{3x} & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & -1 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & -1 & 0 & 1 & 0 \\ 0 & 0 & -1 & 0 & q_{4y} & -q_{4x} & -p_{4y} & p_{4x} & 0 \end{bmatrix} \begin{bmatrix} f_{1x} \\ f_{1y} \\ f_{1x} \\ f_{1y} \\ f_{1x} \\ f_{1y} \\ f_{1x} \\ f_{1y} \\ T_0 \end{bmatrix}$$

$$= \begin{bmatrix} m_2\ddot{g}_{2x}\ \Sigma F_{2x} \\ m_2\ddot{g}_{2y} - \Sigma F_{2y} \\ I_2\ddot{\theta}_2 - \Sigma T_2 - (d_2 * \Sigma F_2) \\ m_3 g_{3y} - \Sigma F_{3y} \\ m_3\ddot{g}_{3x} - \Sigma F_{3x} \\ I_3\ddot{\theta}_3 - \Sigma T_3 - (d_3 * \Sigma F_3) \\ m_4 g_{4x} - \Sigma F_{4x} \\ m_4\ddot{g}_{4y} - \Sigma F_{4y} \\ I_4\ddot{\theta}_4 - \Sigma T_2 - (d_2 * \Sigma F_4) \end{bmatrix} \quad (12)$$

Table 3: Crank Angle Vs Joint Force

I/P ANGLE	Joint Force at Crank Coupler joint	
	X-Component	Y-Component
0	3.74E+03	6.43E+03
36	8.64E+03	6.69E+03
72	3.37E+03	2.73E+03
108	-1.59E+01	1.43E+03
144	-1.89E+03	1.31E+02
180	-2.58E+03	-1.38E+03
216	-2.25E+03	-2.69E+03
252	-1.42E+03	-3.63E+03
288	-7.86E+02	-4.59E+03
324	-1.11E+03	-4.89E+03
360	3.68E+03	6.37E+03

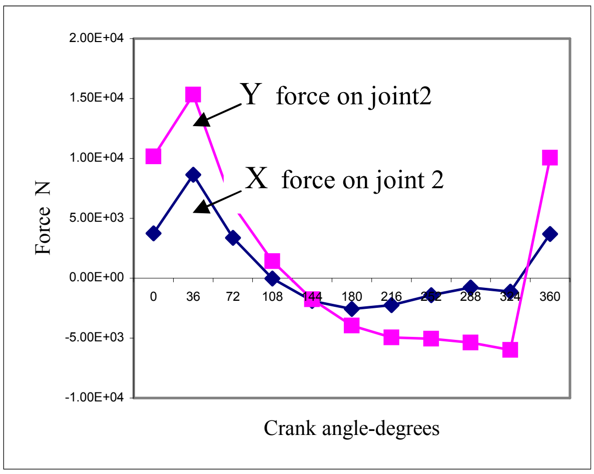

Figure 7: X-Y Component of joint force

Table 3 and Fig. (7) depict the Joint force in X & Y component.

Locus of Journal Center is given by

X-Component = Clearance*Cos(δ)

Y-Component = Clearance*Sin(δ) (14)

Table 4. along with Fig. (8) is the locus of journal center inside the clearance region for the rotation of $0\text{-}360^0$ crank rotation .

Table 4: Crank Angle Vs Center of Journal

I/P ANGLE	Position of Center of Journal (δ)	
	X-Component	Y-Component
0	5.28E-02	9.08E-02
36	8.30E-02	6.43E-02
72	8.16E-02	6.61E-02
108	-1.00E-03	1.05E-01
144	-1.05E-01	7.45E-03
180	-9.26E-02	-4.95E-02
216	-6.75E-02	-8.05E-02
252	-3.83E-02	-9.78E-02
288	-1.79E-02	-1.03E-01
324	-2.35E-02	-1.02E-01
360	5.26E-02	9.09E-02

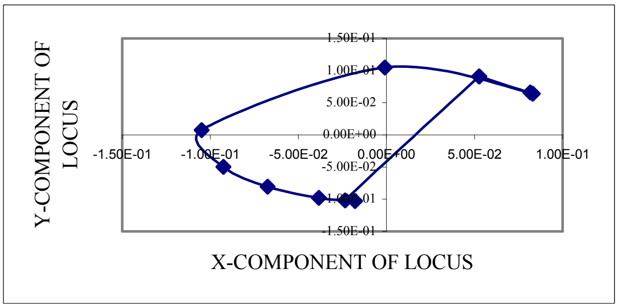

Figure 8: Locus of Journal Center

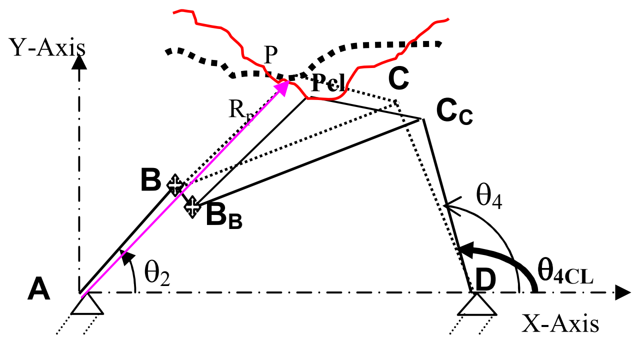

Figure 9: Four bar linkage with & without clearance link

The magnitude of R_p without clearance can be obtained from Fig. (9)

$$R_p = \sqrt{R_{px}^2 + R_{py}^2} \quad (15)$$

The magnitude of $(R_p)_{Mech\ Error}$ with mechanical error is given as Fig. (9) and Eq. (7)

$$(R_p)_{Mech.Error} = R_p + \sqrt{V_{px}^2 + V_{py}^2} \quad (16)$$

Out put angle with clearance can be obtained from Fig. (9) as

$$\Theta_{4CL} = 180 - \tan^{-1}\left\{(bo\text{-}bb)_x / (bo\text{-}bb)_y\right\} \quad (17)$$

5 Results & Conclusions

The influence of joint clearance on mechanical error of coupler point P and error in output angle of four bar mechanism is studied. As crank moves clearance link orientation varies depending upon the joint force as obtained by Eq. (12) which in turn effect the output angle and also the position of coupler point. Table 5. along with Fig. (10) represent various position of point P. The mechanical error is in the range of -0.0566cm to 0.115cm. Table 6. & Fig. (11) show the position of follower angle. The error in output angle with and without clearance is in the range of -0.350^0 to 0.529^0. The figures 11 & 13 show that between 144^0 to 216^0 of crank rotation the error is almost constant . During this range the transmission angles are in the range of -99^0 to -103^0 and for rest of the crank angle the transmission angle is more than 103^0. At crank angles 0^{0}and 360^0 the transmission angle is 147^0 hence the error is maximum. This fact confirms that for optimum transmission angle (90^0 ±50$^{0)}$ the error is minimum [1].

Table 5: Crank Angle Vs Vector R_p Coupler Point

I/P ANGLE	Position of Point P		
	R_p	$(R_p)_{Mech.Error}$	ERROR
0	0.103E+02	0.102E+020	.113E+000
36	0.137E+02	0.138E+02	-0.113E+00
72	0.150E+02	0.151E+02	-0.878E-01
108	0.145E+02	0.146E+02	-0.965E-01
144	0.129E+02	0.130E+02	-0.575E-01
180	0.109E+02	0.110E+02	-0.566E-01
216	0.889E+01	0.897E+01	-0.765E-01
252	0.696E+01	0.706E+01	-0.102E+00
288	0.534E+01	0.544E+01	-0.109E+00
324	0.552E+01	0.553E+01	-0.230E-02
360	0.964E+01	0.953E+01	0.115E+00

Table 6: Crank Angel Vs Follower Position

I/P ANGLE	Without clea.	With clearance.	
	Θ_4	Θ_{4CL}	ERROR
0	-0.780E+02	-0.785E+02	0.431E+00
36	0.881E+02	0.877E+02	0.350E+00
72	-0.889E+02	-0.892E+02	0.281E+00
108	-0.768E+02	-0.770E+02	0.149E+00
144	-0.630E+02	-0.628E+02	-0.162E+00
180	-0.511E+02	-0.509E+02	-0.212E+00
216	-0.431E+02	-0.429E+02	-0.225E+00
252	-0.399E+02	-0.396E+02	-0.242E+00
288	-0.426E+02	-0.424E+02	-0.280E+00
324	-0.544E+02	-0.541E+02	-0.350E+00
360	-0.769E+02	-0.774E+02	0.529E+00

Thus ,this article presents an approach to determine mechanical error of coupler point caused by orientation of clearance link because of joint clearance between crank and coupler link . The problem of calculating the mechanical errors of planar mechanism can be transformed into the problem of velocity analysis of a properly devised sensitivity linkage. The vector representing the mechanical error of coupler point makes, in general an ellipse as the relative position angle of pin varies on clearance circle for fixed crank angle Fig. (5).

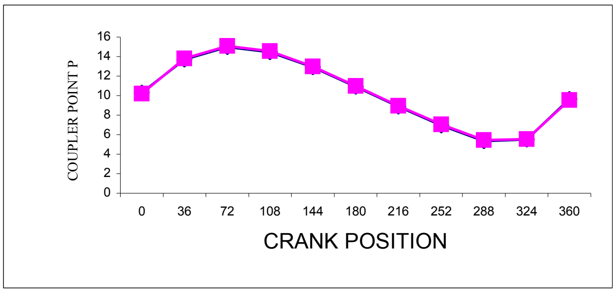

Figure 10:Various position of coupler point P w.r.t. crank angles.

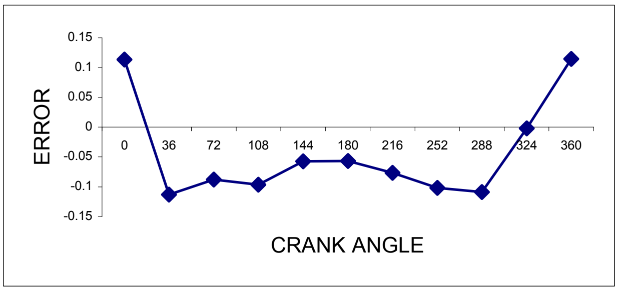

Figure 11: Error in coupler point P with and without clearance and crank angles..

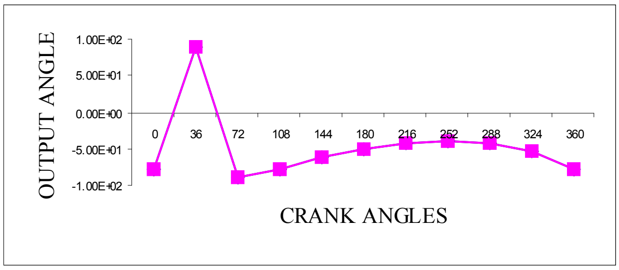

Figure 12: Comparison of output angle with crank angles.

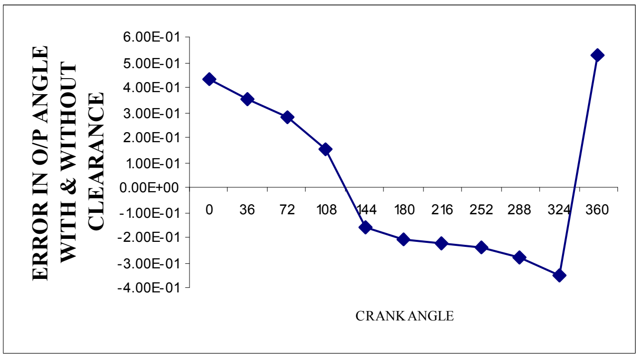

Figure 13 : Error between output angle with and without cleance and crank angles.

Future work

The significant research work have been carried out by researchers on flexible four bar linkages in the area of KEDA, optimization of link parameters etc. by assuming link/s flexible and perfect rigid bearing. Considering bearing as flexible along with linkage/linkages can be carried out in future work.

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