STRUCTURAL SYNTHESIS OF KINEMATIC CHAINS USING EIGENVALUES AND EIGENVECTORS

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Abstract

Several methods are available to detect isomorphism among kinematic chains but each has its own shortcomings. A method based on eigenvalues and eigenvectors of adjacent matrices of kinematic chains is proposed in this paper. This method is reliable and straightforward. Illustrations show the isomorphic test on counter examples.

1 Introduction

One of the most important and challenging problem in structural synthesis of kinematic chains is to identify the possible structural isomorphism between two given chains. Many attempts [1-21] have been made in literature to develop reliable and computationally efficient tests for isomorphism. These tests can be grouped as 1) characteristic polynomial based approaches 2) code-based approaches, 3) Hamming-number based approaches and 4) distance or path based approaches

Uicker and Raicu [2] seem to be the first researchers who have investigated the properties of the characteristic polynomial of the adjacent matrix of a kinematic chain. Murthyunjaya and Raghavan [3] applied Bocher's formula for the determination of the characteristic coefficients and presented a counter example for the uniqueness of the characteristic polynomial and showed that polynomial is unique for closed and connected kinematic graphs. Yan and Hall[4,5] presented rules and theorems by which characteristic polynomial of kinematic chain and its coefficients are determined. Mruthyunjaya and Balasubramanian [6] worked on characteristic polynomial of a vertex-vertex degree matrix, they brought light on counter examples. Dubey and Rao[7] considered characteristic polynomial of distance matrix. Ambekar and Agrawal [8] proposed max code and min code methods for the detection of isomorphism. Kim and Kawk[10] proposed heuristic algorithm that uniquely labels the links of a chain which leads to a unique code. Shin and Krishnamurthy[11] presented the standard code theory for the detection of isomorphism. A.C. Rao[12] introduced the concept of Hamming distances from information and communication theory to the study of kinematic structure. Rao and Varadaraju[13] defined link Hamming string as an index for testing isomorphism. Rao[14] illustrated a method by using chain Hamming matrix by which reliability of isomorphism test based on primary Hamming string is increased. Yan and Hwang[15] defined the linkage path code of a kinematic chain. Yadav[16] presented a sequential three-step test for isomorphism. Vijayananda[17] has developed a new isomorphism test based on the visual description of a chain and is suitable for computer implementation. Shende and Rao[18] proposed a method based on summation polynomials. Zongyu Chang et.al[19]presented a new method based on eigen values and eigen vectors of adjacent matrices of chains . Cubillo and Wan[20] stated the necessary and sufficiency conditions of eigenvalues and eigenvectors to identity isomorphic chains. .Sunkari and Schmidt[21] worked on the reliability and efficiency of the existing spectral methods.

However, most of the methods are based on the adjacent matrices of kinematic chains. Whether a link in a chain is connected directly to any other link of the chain or not is represented in an adjacent matrix ($\boldsymbol{A}$). The elements of adjacent matrix are either zero or one. $\boldsymbol{A}$ is of size nxn for a chain of n links. If link i is directly connected to link j then the element in the i th row and jth column of $\boldsymbol{A}$ is 1, otherwise 0.

2 The Eigenvalues and Eigenvectors Approach

Suppose $\boldsymbol{A}$ and $\mathbf{A}'$ are the adjacent matrices of kinematic chains. $\lambda_1, \lambda_2, \ldots\ldots\lambda_n$ and $\lambda_1', \lambda_2', \ldots\ldots\lambda_n'$ are eigenvalues of $\boldsymbol{A}$ and $\mathbf{A}'$ respectively. $x_1, x_2, \ldots\ldots x_n$ and $x_1', x_2', \ldots\ldots x_n'$ are eigenvectors corresponding to these eigenvalues. With these eigenvectors as column vectors we get two non-singular matrices $\mathbf{X}$ and $\mathbf{X}'$.

$$\mathbf{X} = \{ x_1 , x_2 , \ldots\ldots x_n \} \tag{1}$$

$$\mathbf{X}' = \{ x_1', x_2', \ldots\ldots x_n' \} \tag{2}$$

According to matrix theory,

$$\mathbf{X}^{-1}\mathbf{A}\mathbf{X} = \text{diag}\ \{ \lambda_1, \lambda_2 \ldots\ldots\lambda_n \} \tag{3}$$

$$\mathbf{X}'^{-1}\mathbf{A}'\mathbf{X}' = \text{diag}\ \{ \lambda_1', \lambda_2', \ldots\ldots\lambda_n' \} \tag{4}$$

If two kinematic chains are isomorphic then the eigenvalues of $\boldsymbol{A}$ and $\mathbf{A}'$ are the similar. By interchanging the rows or columns of diagonal matrix diag $\{\lambda_1', \lambda_2', ...\lambda_n'\}$ can be made equal to the diagonal matrix diag $\{\lambda_1, \lambda_2 ....\lambda_n\}$. Therefore there exists a row transformation matrix $\mathbf{T_c}$ such that,

$$\text{diag}\{\lambda_1, \lambda_2 .....\lambda_n\} = \mathbf{T_c}^{-1}\,\text{diag}\{\lambda_1', \lambda_2', ...\lambda_n'\}\,\mathbf{T_c}$$

$$= \mathbf{T_c}^{-1}\ \mathbf{X}'^{-1}\mathbf{A}'\mathbf{X}'\ \mathbf{T_c} \tag{5}$$

$$\text{Let}\ \mathbf{X_n'} = \mathbf{X}'\ \mathbf{T_c} \tag{6}$$

Eq. (5) yields

$$\text{diag}\{\lambda_1, \lambda_2 ....\lambda_n\} = \mathbf{X_n'}^{-1}\ \mathbf{A}'\ \mathbf{X}_n' \tag{7}$$

2.1 Necessary and sufficient conditions for isomorphism of kinetic chains.

Theorem[20]: Suppose $\mathbf{X}$ and $\mathbf{X}'$ are non-singular matrices of eigenvectors of kinematic chains with adjacent matrices A and $\mathbf{A}'$ respectively. Also $\lambda_1, \lambda_2 ....\lambda_n$ and $\lambda_1', \lambda_2', ...\lambda_n'$ are eigenvalues corresponding to eigenvectors of $\mathbf{X}$ and $\mathbf{X}'$.

The necessary and sufficient conditions for isomorphism of kinetic chains is that there is a row transform matrix $\mathbf{T}$ such that,

$$\mathbf{TAT}^{-1} = \mathbf{A}'$$

Necessary condition:

There exist a row transform matrix $\mathbf{T}$ such that Eq. (6) and (7) are true.

Sufficient condition:

$\mathbf{TX}$ is also a feature matrix of $\mathbf{A}'$, that is

$\mathbf{A}'\mathbf{TX} = \mathbf{TX}\ \text{diag}\{\lambda_1, \lambda_2 ....\lambda_n\}$

Proof: From Eq.(7)

$\text{diag}\{\lambda_1, \lambda_2 ....\lambda_n\} = \mathbf{X_n'}^{-1}\mathbf{A}'\mathbf{X_n'}$

i.e, $\text{diag}\{\lambda_1, \lambda_2 .....\lambda_n\} = \text{diag}\{\lambda_1', \lambda_2', ...\lambda_n'\}$

The necessary condition is that if A and $\mathbf{A}'$ are isomorphic then they have same eigenvalues. From sufficient condition

$$\mathbf{A}'\mathbf{TX} = \mathbf{TX}\ \text{diag}\{\lambda_1, \lambda_2 ....\lambda_n\}$$

$$= \mathbf{TX}\ [\mathbf{X}^{-1}\mathbf{AX}]$$

$$\mathbf{A}' = \mathbf{TX[X^{-1}AX]X^{-1}T^{-1}} = \mathbf{TAT}^{-1}$$

Conversely the eigenvalues and eigenvectors of their adjacent matrices are equal. Eigenvectors of adjacent matrix of a chain can be transformed to that of another chain by some row transform matrix $\mathbf{T}$ (which is not symmetrical and self inverse matrix) when chains are isomorphic.

2.1.1 Procedure to identify isomorphic kinematic chains [20].

1) Write down the adjacent matrices of kinematic chains.
2) Compute the eigenvalues and eigenvectors of adjacent matrices.
3) Compare the eigenvalues of adjacent matrices by visual inspection. If the eigenvalues of adjacent matrices are correspondingly equal then go to next step. Otherwise the chains are non-isomorphic.
4) Compare the eigenvectors corresponding to a single eigenvalue of an adjacent matrix with the other to find the interchanging of rows. If the eigenvectors corresponding to single eigenvalue of adjacent matrices are correspondingly equal or prorata, then make the interchanging of rows to one of the matrix and go to next step. Otherwise, the chains are non-isomorphic.
5) Examine the interchanged matrix. If all the vectors of the interchanged matrix are eigenvectors of a corresponding adjacent matrix, then the chains are isomorphic. i.e., we get the row transformation matrix $\mathbf{T}$, which transfers the eigenvectors of an adjacent matrix of a chain to that of another chain. Otherwise the chains are non-isomorphic. The procedure is illustrated more clearly by the following examples.

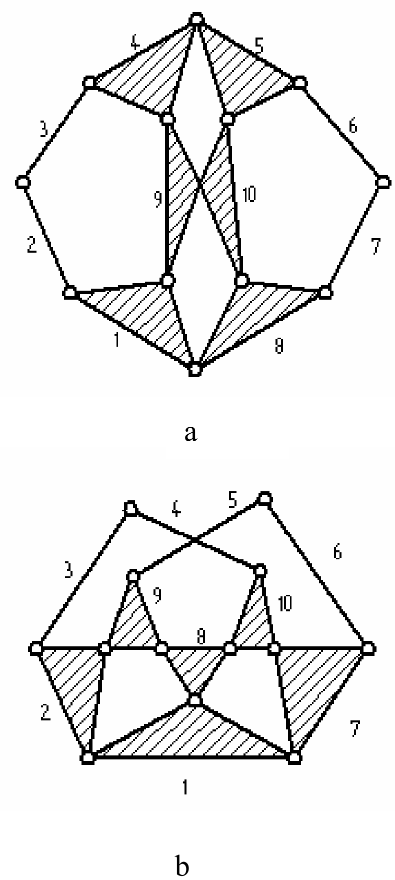


Figure 1: Ten link, single degree of freedom, non-isomorphic kinematic chains

Adjacent matrices of kinematic chains shown in

Fig.(1)a & b are,

$$A=\begin{bmatrix}0&1&0&0&0&0&0&1&1&0\\1&0&1&0&0&0&0&0&0&0\\0&1&0&1&0&0&0&0&0&0\\0&0&1&0&1&0&0&0&1&0\\0&0&0&1&0&1&0&0&0&1\\0&0&0&0&1&0&1&0&0&0\\0&0&0&0&0&1&0&1&0&0\\1&0&0&0&0&0&1&0&0&1\\1&0&0&1&0&0&0&0&0&1\\0&0&0&0&1&0&0&1&1&0\end{bmatrix}$$

$$B=\begin{bmatrix}0&1&0&0&0&0&1&1&0&0\\1&0&1&0&0&0&0&0&1&0\\0&1&0&1&0&0&0&0&0&0\\0&0&1&0&0&0&0&0&0&1\\0&0&0&0&0&1&0&0&1&0\\0&0&0&0&1&0&1&0&0&0\\1&0&0&0&0&1&0&0&0&1\\1&0&0&0&0&0&0&0&1&1\\0&1&0&0&1&0&0&1&0&0\\0&0&0&1&0&0&1&1&0&0\end{bmatrix}$$

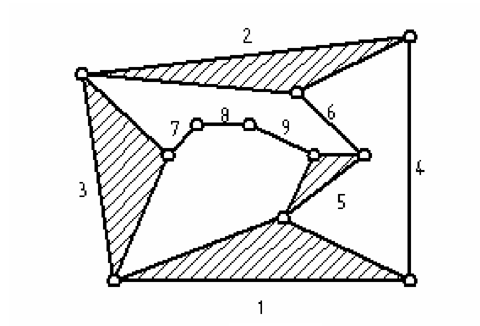
Fig (1) shows two ten link kinematic chains whose chain-Hamming strings are identical. Hence, they were classified as isomorphic [13].

MATLAB ver.7.2 is used to evaluate the eigenvalues and the eigenvectors of adjacent matrices. The eigenvalues of the two adjacent matrices of ***A*** and ***B*** sorted in the order of the magnitude are

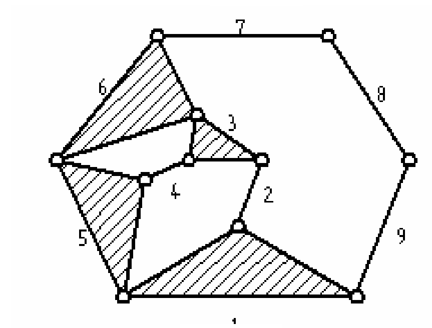
$[-2.5616,-2.0,-1.4142,-0.7321, 0.0, 0.0, 1.0, 1.4142, 1.5616, 2.7321]^T$

$[-2.7321,-1.4142,-1.4142,-1.0,-0.7321, 0.7321, 1.0, 1.4142, 1.4142, 2.7321]^T$

Since the eigenvalues are distinct the chains are non-isomorphic.



a



b

Figure 2: Nine link, two degree of freedom, isomorphic kinematic chains

Adjacent matrices of kinematic chains shown in Fig. (2) a &b are,

$$A=\begin{bmatrix}0&0&1&1&1&0&0&0&0\\0&0&1&1&0&1&0&0&0\\1&1&0&0&0&0&1&0&0\\1&1&0&0&0&0&0&0&0\\1&0&0&0&0&1&0&0&1\\0&1&0&0&1&0&0&0&0\\0&0&1&0&0&0&0&1&0\\0&0&0&0&0&0&1&0&1\\0&0&0&0&1&0&0&1&0\end{bmatrix}$$

$$B=\begin{bmatrix}0&1&0&0&1&0&0&0&1\\1&0&1&0&0&0&0&0&0\\0&1&0&1&0&1&0&0&0\\0&0&1&0&1&0&0&0&0\\1&0&0&1&0&1&0&0&0\\0&0&1&0&1&0&1&0&0\\0&0&0&0&0&1&0&1&0\\0&0&0&0&0&0&1&0&1\\1&0&0&0&0&0&0&1&0\end{bmatrix}$$

Table 1: Eigenvalues and eigenvectors of ***A***

-2.3585	-1.9076	-1.2927	-0.5739	0.1994	0.6298	1.2527	1.4834	2.5674
0.4766	-0.1047	0.3822	-0.1358	-0.1152	0.6106	-0.0552	-0.1341	0.4361
0.3842	0.4369	0.0140	0.2920	0.2442	-0.4808	0.0842	-0.3316	0.4102
-0.4917	-0.1537	0.0869	0.4461	-0.3826	0.0490	0.4403	-0.0979	0.4193
-0.3650	-0.1741	-0.3065	-0.2722	0.6467	0.2061	0.0231	-0.3139	0.3296
-0.2675	0.5275	-0.2745	-0.0960	-0.2871	0.1295	-0.5326	0.2129	0.3706
-0.0495	-0.5055	0.2015	-0.3415	-0.2155	-0.5579	-0.3580	-0.0800	0.3041
0.2988	-0.0390	-0.5085	-0.4122	-0.2053	-0.0989	0.5226	0.3204	0.2303
-0.2131	0.2281	0.5704	-0.2096	0.3416	-0.1113	0.2144	0.5733	0.1720
0.2038	-0.3961	-0.2289	0.5325	0.2734	0.0288	-0.2541	0.5300	0.2114

Table 2: Eigenvalues and eigenvectors of ***B***

-2.3585	-1.9076	-1.2927	-0.5739	0.1994	0.6298	1.2527	1.4834	2.5674
-0.2675	0.5275	-0.2745	-0.0960	-0.2871	-0.1295	-0.5326	-0.2129	0.3706
-0.0495	-0.5055	0.2015	-0.3415	-0.2155	-0.5579	-0.3580	0.0800	0.3041
0.3842	0.4369	0.0140	0.2920	0.2442	-0.4808	0.0842	0.3316	0.4102
-0.3650	-0.1741	-0.3065	-0.2722	0.6467	0.2061	0.0231	0.3139	0.3296
0.4766	-0.1047	0.3822	-0.1358	-0.1152	0.6106	-0.0552	0.1341	0.4361
-0.4917	-0.1537	0.0869	0.4461	-0.3826	0.0490	0.4403	0.0979	0.4193
0.2988	-0.0390	-0.5085	-0.4122	-0.2053	-0.0989	0.5226	-0.3204	0.2303
-0.2131	0.2281	0.5704	-0.2096	0.3416	-0.1113	0.2144	-0.5733	0.1720
0.2038	-0.3961	-0.2289	0.5325	0.2734	0.0288	-0.2541	-0.5300	0.2114

From the Table 1 and 2 (in which the first rows are eigenvalues) it is observed that the two adjacent matrices have the same eigenvalues.

Comparing the eigenvalues corresponding to a single eigenvalue which are correspondingly equal or prorata of one chain with the other, we find the correspondence between the links given by, 1↔5, 2↔3, 3↔6, 4↔4,5↔1, 6↔2, 7↔7, 8↔8, 9↔9.

The eigenvectors of an adjacent matrix of a chain can be transformed to that of another chain by some row transform matrix $\mathbf{T}$ when the chains are isomorphic.

$\mathbf{T}(i,j)$ is the row transformation matrix which interchanges the i^{th} &j^{th} row of $\boldsymbol{A}$ when it is left multiplied by $\boldsymbol{A}$.

For example,

$$\mathbf{T}(1,5)=\begin{bmatrix} 0&0&0&0&1&0&0&0&0\\ 0&1&0&0&0&0&0&0&0\\ 0&0&1&0&0&0&0&0&0\\ 0&0&0&1&0&0&0&0&0\\ 1&0&0&0&0&0&0&0&0\\ 0&0&0&0&0&1&0&0&0\\ 0&0&0&0&0&0&1&0&0\\ 0&0&0&0&0&0&0&1&0\\ 0&0&0&0&0&0&0&0&1 \end{bmatrix}$$

Then,

$$\mathbf{T}=\mathbf{T}(1,5)\ \mathbf{T}(2,3)\ \mathbf{T}\ (3,6)=$$

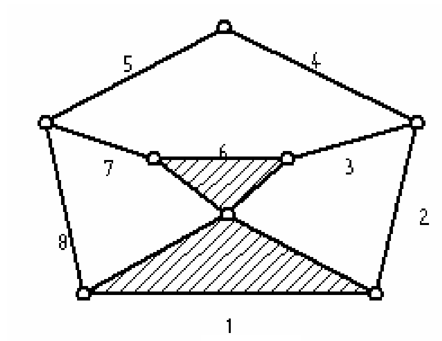
$$\begin{bmatrix} 0&0&0&0&1&0&0&0&0\\ 0&0&0&0&0&1&0&0&0\\ 0&1&0&0&0&0&0&0&0\\ 0&0&0&1&0&0&0&0&0\\ 1&0&0&0&0&0&0&0&0\\ 0&0&1&0&0&0&0&0&0\\ 0&0&0&0&0&0&1&0&0\\ 0&0&0&0&0&0&0&1&0\\ 0&0&0&0&0&0&0&0&1 \end{bmatrix}$$

$$\mathbf{T}^{-1}=\begin{bmatrix} 0&0&0&0&1&0&0&0&0\\ 0&0&1&0&0&0&0&0&0\\ 0&0&0&0&0&1&0&0&0\\ 0&0&0&1&0&0&0&0&0\\ 1&0&0&0&0&0&0&0&0\\ 0&1&0&0&0&0&0&0&0\\ 0&0&0&0&0&0&1&0&0\\ 0&0&0&0&0&0&0&1&0\\ 0&0&0&0&0&0&0&0&1 \end{bmatrix}$$

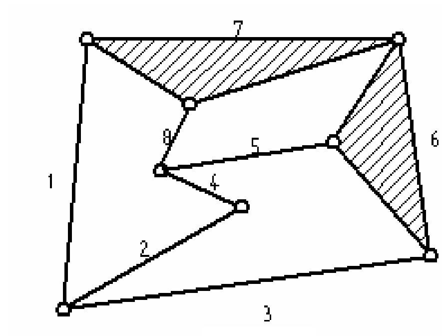
Which makes,

$$\mathbf{TAT}^{-1}=\mathbf{B}$$

Therefore, the two chains are isomorphic



a



b

Figure 3: Eight link, single degree of freedom, isomorphic kinematic chains

Adjacent matrices of kinematic chains shown in Fig.(3)a &b are,

$$A=\begin{bmatrix}0&1&0&0&0&1&0&1\\1&0&1&1&0&0&0&0\\0&1&0&1&0&1&0&0\\0&1&1&0&1&0&0&0\\0&0&0&1&0&0&1&1\\1&0&1&0&0&0&1&0\\0&0&0&0&1&1&0&1\\1&0&0&0&1&0&1&0\end{bmatrix}$$

$$B=\begin{bmatrix}0&1&1&0&0&0&1&0\\1&0&1&1&0&0&0&0\\1&1&0&0&0&1&0&0\\0&1&0&0&1&0&0&1\\0&0&0&1&0&1&0&1\\0&0&1&0&1&0&1&0\\1&0&0&0&0&1&0&1\\0&0&0&1&1&0&1&0\end{bmatrix}$$

The eigenvalues and eigenvectors of adjacent matrices are shown in Tables 3 and 4, the first row indicates the eigenvalues. Chains A and B have same eigenvalues and different eigenvectors in which eigenvalues $\lambda_3 = \lambda_4 = -1.0$ are complex. From the eigenvectors of complex eigenvalues, we cannot find correspondence between the links.

Comparing the eigenvectors corresponding to a single eigenvalue of one chain with the other we find the correspondence between the links given by 1↔6, 2↔5, 3↔8, 4↔4, 5↔2, 6↔7, 7↔1, 8↔3. Computing row transformation matrix.

$$\mathbf{T} = \mathbf{T}(1,6)\ \mathbf{T}(2,5)\ \mathbf{T}(3,8)\ \mathbf{T}(6,7)$$
$$=\begin{bmatrix}0&0&0&0&0&0&1&0\\0&0&0&0&1&0&0&0\\0&0&0&0&0&0&0&1\\0&0&0&1&0&0&0&0\\0&1&0&0&0&0&0&0\\1&0&0&0&0&0&0&0\\0&0&0&0&0&1&0&0\\0&0&1&0&0&0&0&0\end{bmatrix}$$

Which makes

$$\mathbf{B} = \mathbf{T}\ \mathbf{A}\ \mathbf{T}^{-1}$$

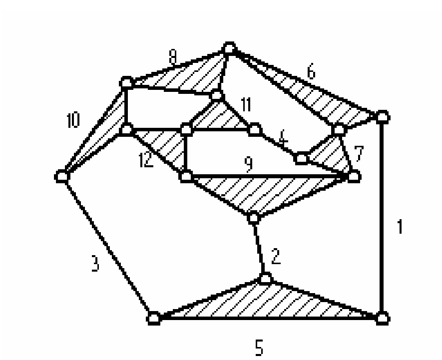
Therefore chains are isomorphic.

Table 3: Eigenvalues and eigenvectors of ***A***

-2.4142	-1.7321	-1.000	-1.000	0.4142	1.000	1.7321	3.000
0.5000	0.0000	-0.2000	0.2915	-0.5000	0.5000	0.0000	-0.3536
-0.3536	-0.2299	-0.2123	-0.5744	-0.3536	-0.0000	0.4440	-0.3536
0.3536	-0.2299	0.6123	-0.0087	0.3536	0.0000	0.4440	-0.3536
-0.0000	0.6280	-0.2000	0.2915	-0.0000	-0.5000	0.3251	-0.3536
-0.0000	-0.6280	-0.2000	0.2915	-0.0000	-0.5000	-0.3251	-0.3536
-0.5000	-0.0000	-0.2000	0.2915	0.5000	0.5000	-0.0000	-0.3536
0.3536	0.2299	-0.2123	-0.5744	0.3536	-0.0000	-0.4440	-0.3536
-0.3536	0.2299	0.6123	-0.0087	-0.3536	0.0000	-0.4440	-0.3536

Table 4: Eigenvalues and eigenvectors of ***B***

-2.4142	-1.7321	-1.000	-1.000	0.4142	1.000	1.7321	3.000
0.3536	0.2299	-0.5935	0.1510	0.3536	-0.0000	-0.4440	0.3536
0.0000	-0.6280	0.2690	0.2294	-0.0000	-0.5000	-0.3251	0.3536
-0.3536	0.2299	0.0555	-0.6099	-0.3536	0.0000	-0.4440	0.3536
-0.0000	0.6280	0.2690	0.2294	-0.0000	-0.5000	0.3251	0.3536
-0.3536	-0.2299	-0.5935	0.1510	-0.3536	0.0000	0.4440	0.3536
0.5000	-0.0000	0.2690	0.2294	-0.5000	0.5000	0.0000	0.3536
-0.5000	-0.0000	0.2690	0.2294	0.5000	0.5000	0.0000	0.3536
0.3536	-0.2299	0.0555	-0.6099	0.3536	-0.0000	0.4440	0.3536



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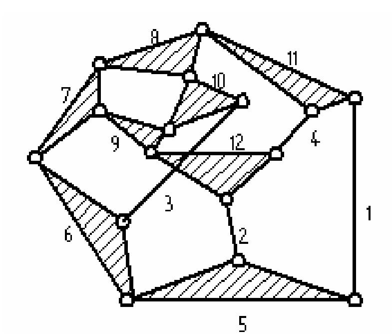

b

Figure 4: Twelve link, single degree of freedom, non-isomorphic kinematic chains

The adjacent matrices of kinematic chains shown in Fig. (4) a&b are,

$$A=\begin{bmatrix}0&0&0&0&1&1&0&0&0&0&0&0\\0&0&0&0&1&0&0&0&1&0&0&0\\0&0&0&0&1&0&0&0&0&1&0&0\\0&0&0&0&0&0&1&0&0&0&1&0\\1&1&1&0&0&0&0&0&0&0&0&0\\1&0&0&0&0&0&1&1&0&0&0&0\\0&0&0&1&0&1&0&0&1&0&0&0\\0&0&0&0&0&1&0&0&0&1&1&0\\0&1&0&0&0&0&1&0&0&0&0&1\\0&0&1&0&0&0&0&1&0&0&0&1\\0&0&0&1&0&0&0&1&0&0&0&1\\0&0&0&0&0&0&0&0&1&1&1&0\end{bmatrix}$$

$$B=\begin{bmatrix}0&0&0&0&1&0&0&0&0&0&1&0\\0&0&0&0&1&0&0&0&0&0&0&1\\0&0&0&0&0&1&0&0&0&1&0&0\\0&0&0&0&0&0&0&0&0&0&1&1\\1&1&0&0&0&1&0&0&0&0&0&0\\0&0&1&0&1&0&1&0&0&0&0&0\\0&0&0&0&0&1&0&1&1&0&0&0\\0&0&0&0&0&0&1&0&0&1&1&0\\0&0&0&0&0&0&1&0&0&1&0&1\\0&0&1&0&0&0&0&1&1&0&0&0\\1&0&0&1&0&0&0&1&0&0&0&0\\0&1&0&1&0&0&0&0&1&0&0&0\end{bmatrix}$$

The kinematic chains shown in Fig. (4) have identical characteristic polynomial coefficients hence, characteristic polynomial approach fails [19]

The eigenvalues and eigenvectors of adjacent matrices are shown in Tables 5 and 6. Chains *A* and *B* have the same eigenvalues but different eigenvectors.

Comparing the eigenvectors corresponding to a single eigenvalue of one chain with the other, there is no correspondence between the links therefore transformation matrix $\mathbf{T}$ does not exist, hence $\mathbf{B} \neq \mathbf{TAT^{-1}}$. Therefore the chains are non-isomorphic.

Table 5: Eigenvalues and eigenvectors of ***A***

-2.6264	-2.0000	-1.5382	-1.4142	-0.9815	0.0000	0.4150	0.6350	1.4142	1.5713	1.7668	2.7580
0.2192	0.2236	0.2493	0.3536	-0.2744	-0.5000	0.2770	0.0214	0.3536	0.1231	0.3693	-0.1892
0.2192	0.2236	0.2493	-0.3536	-0.2744	0.5000	0.2770	0.0214	-0.3536	0.1231	0.3693	-0.1892
0.2445	0.0000	0.4127	-0.0000	0.4518	-0.0000	-0.5160	0.1683	0.0000	-0.3684	0.3114	-0.1962
0.0260	-0.4472	0.1927	-0.0000	-0.4066	-0.0000	-0.2387	0.5947	-0.0000	0.2630	-0.2433	-0.2406
-0.2600	-0.2236	-0.5924	0.0000	0.0988	0.0000	0.0914	0.3326	0.0000	-0.0778	0.5943	-0.2084
-0.3157	-0.2236	0.2090	-0.5000	0.1705	0.0000	0.0235	-0.3190	0.5000	0.2712	0.0582	-0.3135
0.2305	0.4472	-0.3970	0.0000	0.0668	-0.0000	-0.4618	-0.0682	-0.0000	0.5126	-0.0718	-0.3146
0.3795	-0.2236	-0.1737	0.3536	0.0402	0.5000	0.1946	-0.1558	0.3536	-0.2095	-0.1947	-0.3609
-0.3157	-0.2236	0.2090	0.5000	0.1705	-0.0000	0.0235	-0.3190	-0.5000	0.2712	0.0582	-0.3135
-0.3821	0.2236	-0.0424	0.0000	-0.5422	-0.0000	-0.3055	-0.2257	0.0000	-0.5012	-0.0441	-0.3329
-0.2989	0.4472	0.1007	-0.0000	0.3323	0.0000	0.3628	0.4458	0.0000	-0.0993	-0.3580	-0.3489
0.3795	-0.2236	-0.1737	-0.3536	0.0402	-0.5000	0.1946	-0.1558	-0.3536	-0.2095	-0.1947	-0.3609

Table 6: Eigenvalues and eigenvectors of **B**

-2.6264	-2.0000	-1.5382	-1.4142	-0.9815	0.0000	0.4150	0.6350	1.4142	1.5713	1.7668	2.7580
-0.2192	0.2236	0.2493	-0.3536	0.2744	0.5000	0.2770	0.0214	0.3536	-0.1231	0.3693	-0.1892
-0.2192	0.2236	0.2493	0.3536	0.2744	-0.5000	0.2770	0.0214	-0.3536	-0.1231	0.3693	-0.1892
0.0260	-0.4472	0.1927	0.0000	0.4066	0.0000	-0.2387	0.5947	-0.0000	-0.2630	-0.2433	-0.2406
-0.2445	0.0000	0.4127	0.0000	-0.4518	-0.0000	-0.5160	0.1683	-0.0000	0.3684	0.3114	-0.1962
0.2547	-0.4472	-0.0660	-0.0000	-0.4910	-0.0000	0.2220	-0.0398	0.0000	-0.4829	0.3774	-0.2513
-0.2305	0.4472	-0.3970	-0.0000	-0.0668	-0.0000	-0.4618	-0.0682	0.0000	-0.5126	-0.0718	-0.3146
0.3767	0.0000	0.4840	0.0000	0.1500	0.0000	-0.1749	-0.5982	-0.0000	-0.0595	-0.2610	-0.3758
-0.3795	-0.2236	-0.1737	-0.3536	-0.0402	-0.5000	0.1946	-0.1558	0.3536	0.2095	-0.1947	-0.3609
-0.3795	-0.2236	-0.1737	0.3536	-0.0402	0.5000	0.1946	-0.1558	-0.3536	0.2095	-0.1947	-0.3609
0.2989	0.4472	0.1007	0.0000	-0.3323	0.0000	0.3628	0.4458	-0.0000	0.0993	-0.3580	-0.3489
0.3210	-0.0000	-0.3174	0.5000	0.2217	0.0000	-0.1071	0.0534	0.5000	0.2895	0.2751	-0.2706
0.3210	0.0000	-0.3174	-0.5000	0.2217	-0.0000	-0.1071	0.0534	-0.5000	0.2895	0.2751	-0.2706

Conclusion

This method based on eigenvalues and eigenvectors is reliable and computationally efficient to detect isomorphism. Eigenvalues of adjacent matrices of two chains are equal when the chains are isomorphic. In case of the same eigenvalues, a row transformation matrix can transform eigenvectors of an adjacent matrix of a chain to that of another chain when the two chains are isomorphic. Several examples are provided to demonstrate this theory.

References:

[1] E.R.Tuttle, "Generation of planar kinematic chains", *Mech. Mach. Theory*, Vol. 31, (1996) pp. 729-748.

[2] J.J Uicker.and A.Raicu., "A method for the identification and recognition of equivalence of kinematic chains" *Mech. Mach. Theory*, Vol. 10, 1975, pp. 375-383.

[3] T. S. Mruthyunjaya, and M.R.Raghavan, "Structural analysis of kinematic chains and mechanisms based on matrix representation", *ASME, J.Mech. Design*, Vol. 101, No 3, 1979 pp. 488-494.

[4] H.S.Yan and A.S.Hall, "Linkage characteristic polynomials : definition, coefficients by inspection" *J. Mech. Design*, ASME Trans,Vol. 103, 1981, pp. 578-584.

[5] H.S.Yan and A.S.Hall, "Linkage characteristic polynomials assembly theoremsuniqueness", *J. Mech. Design, ASME Tarns*,Vol. 104, 1982, pp. 11-20.

[6] T.S.Mruthyunjaya and H.R. Balasubramanian, "In quest of a reliable and efficient computational test for detection of isomorphism in kinematic chains", *Mech. Mach. Theory*, Vol. 22, 1987, pp. 131-139.

[7] R.K.Dubey and A.C.Rao, "New characteristic polynomial A reliable index to detect isomorphism between kinematic chains in : Proceedings of the National

Conference on Machine and Mechanism", IISc, Bangalore, 1985, pp. 36-40.

[8] A.G.Ambekar and V.P.Agrawal, "On canonical numbering of kinematic chains and isomorphism problem : Max code", *ASME paper no 86-DET-169*.

[9] A.G.Ambekar and V.P.Agrawal, "On canonical numbering of kinematic chains and isomorphism problem : Min code", *Mech. Mach. Theory*, Vol. 22, 1987, pp. 453-461.

[10] J.K.Kim and B.M.Kwak, , "An algorithm of topological ordering for unique representation of graphs", *J. Mech. Design*, *ASME Trans*,Vol. 114, 1992. pp. 103-108.

[11] J.K.Shin and S.Krishnamurty, "On identification and canonical numbering of pin jointed kinematic chains", *J.Mech. Design, ASME Trans*,Vol. 116, 1994, pp. 182-188.

[12] A.C.Rao,"Comparison of plane and spatial kinematic chains for structural error performance using pseudo-Hamming distances",*Indian J.Tech*., Vol.26,1988, pp.155-160.

[13] A.C.Rao and D.Varada Raju, "Application of the Hamming number technique to detect isomorphism among kinematic chains and inversions", *Mech. Mach. Theory*, Vol. 26, 1991 pp. 55-75.

[14] A.C.Rao, "Hamming number technique I: Further applications", *Mech. Mach. Theory*, Vol. 32, 1997, PP 477-488.

[15] H.S.Yan and W.M.Hwang, "Linkage Path code", *Mech.* Mach. *Theory*, Vol.19, 1984 pp. 425-429.

[16] J.N.Yadav, C.R.Pratap and V.P.Agrawal, "Detection of isomorphism among kinematic chains using the distance concept", *J. Mech. Design, ASME Trans*, Vol.117, 1995, pp. 607-611.

[17] K.Vijayananda, "Computer aided structural synthesis of linkages and epicyclic gear transmissions", Ph.D. thesis I.I.Sc, Bangalore, 1994.

[18] S.Shende and A.C.Rao, "Isomorphism in kinematic chains", *Mech. Mach. Theory*, Vol. 29, 1994, pp. 1065-1070.

[19] Zongyu Chang,Ce Zhang, Yuhu Yang,Yuxin Wang, " A new method to mechanism kinematic chain isomorphism identification.", *Mech and Mach Theory*, Vol. 37, 2000, pp. 411-417.

[20] J. P. Cubillo and Jinbao Wan. "Comments on mechanism kinematics chain isomorphism identification using adjacent matrices." *Mech and Mach Theory*, Vol. 40, 2004, pp. 131-139.

[21] R. P Sunkari and L.C. Schmidt. " Reliability and efficiency of the existing spectral methods for isomorphism detection." *J. Mech. Design*, *ASME Trans*,Vol. 128, Nov. 2006, pp. 1246-1252.