Minimizing Cable Swing in a Gantry Crane Using the IDA-PBC Methodology

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Abstract

Precise payload positioning by an overhead crane is difficult due to the fact that the payload can exhibit a pendulum-like swinging motion. The stabilization of loads that are carried by cranes is tedious, and the lack of truly efficient control strategies implies a large economical loss due to the additional time involved in the process. From a control theoretical point of view, cranes are underactuated mechanical systems which give rise to challenging control issues. Motivated by the desire to achieve fast and precise payload positioning while minimizing swinging motion, several researchers have developed various controllers for overhead crane systems. In this paper, we apply a controller design technique called interconnection and damping assignment-passivity based control (IDA-PBC), that achieves stabilization for underactuated mechanical systems invoking the physically motivated principles of energy shaping and damping injection. IDA-PBC endows the closed-loop system with a Hamiltonian structure with a desired energy function that qualifies as a Lyapunov function for the desired equilibrium. The success of this method relies on the possibility of solving a set of partial differential equations (PDEs) that identify the energy functions that can be assigned to the closed-loop. In this paper, we use a partial feedback-linearization inner-loop for explicit solution of these PDEs.

Keywords: IDA-PBC, underactuated systems, cable-operated robot

1 Introduction

Gantry cranes are all pervasive in heavy engineering industry. A schematic representative of one such mechanism operating in two dimensions is shown in Figure 1. The objective is point to point positioning of the payload with minimum cable swing. There are two actuators- a linear actuator which actuates the cart and a rotary one which actuates the winch. For the purpose of the study here, we make the following assumptions:

1. The cable is massless and inelastic.

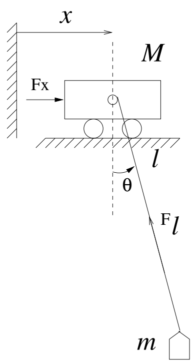

Figure 1: Overhead gantry crane

2. Dissipative forces on the cart and at the winch are negligible.

Assumption 1 simplifies the dynamic model and is a reasonable assumption given the comparatively large inertias of the payload and the cart. Assumption 2 is used to simplify the modeling. A more general mechanism would involve two translational motions of the cart and a spherical pendulum-like motion of the payload.

Several researchers have examined the control problem for the overhead crane system to achieve precise payload positioning with minimum swing. Fang *et al* [1] utilize a simple proportional-derivative (PD) controller to asymptotically regulate the overhead crane system, the coupling between the planar gantry position and the payload angle is increased by the nonlinear controllers. In [2], an overhead crane that exhibits double-pendulum dynamics is investigated by Weiping *et al.*

Control of mechanical systems in a nonlinear setting has received much attention in the past decade. Amongst the techniques developed, a general and promising one has been the IDA-PBC methodology. The idea here is to synthesize a controller that stabilizes the closed loop system about a desired equilibrium and imparts certain characteristics to the closed loop response. In [3] and [4], passivity based interconnection and damping assignment control techniques are used to stabilize underactuated mechanical systems. The asymptotic stabilization of classical ball and beam system and a novel inertia wheel pendulum is achieved through a new parametrization of the closed loop inertia matrix. The matching conditions of controlled Lagrangian and IDA-PBC are discussed in [5]. The IDA-PBC methodology is extended to the class of underactuated mechanical systems with kinematic constraints in [6]. It also introduces the simplified matching equations on constrained manifold. This work is closely related to the controlled Lagrangian strategy for generalized matching equations for underactuated mechanical systems proposed in [7] and [8]. In [9], Kenji Fujimoto *et al* presented an asymptotic stabilization procedure of nonholonomic systems which are described in Hamiltonian framework. These systems are then transformed into canonical forms with specified structure matrices using generalized canonical transformations. In [10] Sorensen *et al* propose augmentation of kinematic inputs with standard Hamiltonian formulation. These inputs change the internal structure of the mechanical system but do not change the stored total energy of the system. Potential energy shaping based controller for the point to point control of a gantry crane is discussed in [11] wherein pulley dynamics is modeled as a holonomic constraint. This concept is further extended in [12] for a combined flatness and energy based controller design which can robustly track an off-line computed trajectory.

The paper is organized as follows: Section 2 presents the dynamic model of the crane in port-Hamiltonian framework begining with the Euler-Lagrange equations of motion. We then perform partial feedback linearization since it facilitates the controller design using IDA-PBC methodology. Section 3 starts with a brief introduction to the IDA-PBC theory applied to such systems. We then discuss the controller design strategy based on IDA-PBC methodology to gantry crane problem. Simulation results are discussed in Section 4. Finally, we wrap up this paper with some concluding remarks in Section 5.

2 Dynamic Model

In this section we develop the Euler-Lagrange equations of motion for the overhead gantry crane system. The configuration variables are

$$q = [\theta \quad x \quad l]^T \quad (= [q_1 \quad q_2 \quad q_3]^T)$$

where $\theta \in S^1$ denotes the payload angle about the vertical axis, $x \in \mathbb{R}$ denotes the gantry position along the X coordinate axis and $l \in \mathbb{R}$ denotes the cable length. The control $u \in \mathbb{R}^2$ is defined as

$$u = [F_x \quad F_l]^T \tag{1}$$

where F_x and F_l represent the control-force inputs acting on the cart and the pulley, respectively. Note that the rotary actuation of the winch is translated as a tensile force on the cable for the purpose of control design. The control objective is to move the payload from any position to the desired position specified as:

$$q_D = [0 \quad x_D \quad l_D]^T.$$

Notice that the desired configuration is a stable equilibrium and the control challenge is more of improving the transient performance in a large domain of attraction. The Lagrangian for the overhead gantry crane can be expressed as,

$$\mathcal{L} = \frac{1}{2}\dot{q}^T M(q)\dot{q} - V(q) \tag{2}$$

where,

$$M(q) = \begin{bmatrix} ml^2 & ml\cos\theta & 0 \\ ml\cos\theta & (M+m) & m\sin\theta \\ 0 & m\sin\theta & m \end{bmatrix},$$

and

$$V(q) = -mgl\cos\theta$$

where M is the mass of the cart, m is the mass of the payload. Here $M(q)$ is the mass matrix and $V(q)$ represents the potential energy of the system. The resulting Euler-Lagrange equations are,

$$\begin{aligned}
0 &= ml^2\ddot{\theta} + ml\cos\theta\ddot{x} + 2ml\dot{\theta}\dot{l} + mgl\sin\theta \\
F_x &= ml\ddot{\theta}\cos\theta + (M+m)\ddot{x} + m\ddot{l}\sin\theta + 2m\dot{l}\dot{\theta}\cos\theta \\
&\quad -ml\dot{\theta}^2\sin\theta \\
F_l &= m\sin\theta\ddot{x} + m\ddot{l} - ml\dot{\theta}^2 - mg\cos\theta.
\end{aligned}$$

These Euler-Lagrange equations can be cast in the following form

$$M(q)\ddot{q} + C(q,\dot{q})\dot{q} + \nabla V(q) = Gu \tag{3}$$

where $C(q,\dot{q})\dot{q}$, represent the centripetal-Coriolis terms. Before proceeding with the controller design, we partially feedback linearize the system since this facilitates the design of the IDA-PBC controller [5, 13] and [14].

2.1 Partial Feedback Linearization

We proceed as follows. With the vector $q \in \mathbb{R}^n$ of generalized coordinates partitioned as $q_1 \in R^{n-m}$ and $q_a \in \mathbb{R}^m$, we may write the dynamic equations of the n degrees of freedom system as,

$$\begin{aligned}
m_{11}\ddot{q}_1 + {m_{12}}^T\ddot{q}_a + f_1 &= 0 \qquad (4)\\
m_{12}\ddot{q}_1 + m_{22}\ddot{q}_a + f_2 &= u, \qquad (5)
\end{aligned}$$

where,

$$M = \begin{bmatrix} m_{11} & {m_{12}}^T \\ m_{12} & m_{22} \end{bmatrix} \tag{6}$$

is a partition of the symmetric, positive definite inertia matrix, the vector functions f_1 and f_2 contain Coriolis-centrifugal and gravitational terms. Note that $q_a = [q_2 \quad q_3]^T$ represents actuated co-ordinates. For notational simplicity we will henceforth not write the explicit dependence on q of these coefficients.

Let $v \triangleq {m_{22}}^{-1}(u - f_2 - m_{12}\ddot{q}_1)$ denote the new input and we have a partially linearized system given by

$$\begin{bmatrix} \ddot{q}_1 \\ \ddot{q}_a \end{bmatrix} = \begin{bmatrix} -{m_{11}}^{-1}f_1 \\ 0 \end{bmatrix} + \begin{bmatrix} -{m_{11}}^{-1}{m_{12}}^T \\ I \end{bmatrix} v. \tag{7}$$

Recasting in terms of momentum coordinates to facilitate a Hamiltonian description we have

$$\dot{q} = p_f \tag{8}$$

$$\dot{p}_f = \begin{bmatrix} -m_{11}{}^{-1}f_1 \\ 0 \end{bmatrix} + \begin{bmatrix} -m_{11}{}^{-1}m_{12}{}^{T} \\ I \end{bmatrix} v, \tag{9}$$

where p_f represents momenta after feedback linearization. In explicit form (9) yields

$$\begin{aligned} \dot{q} &= p_f \\ \dot{p}_f &= \begin{bmatrix} -\frac{1}{q_3}(2\dot{q}_3\dot{q}_1 + g\sin q_1) \\ 0 \\ 0 \end{bmatrix} + \begin{bmatrix} -\frac{\cos q_1}{q_3} & 0 \\ 1 & 0 \\ 0 & 1 \end{bmatrix} v. \end{aligned}$$

For future notational convenience we define

$$\begin{aligned} B_2 &\triangleq \begin{bmatrix} -\frac{\cos q_1}{q_3} & 0 \\ 1 & 0 \\ 0 & 1 \end{bmatrix}, \text{ and} \\ F_2(q,p_f) &\triangleq \begin{bmatrix} -\frac{1}{q_3}(2\dot{q}_3\dot{q}_1 + g\sin q_1) \\ 0 \\ 0 \end{bmatrix}. \end{aligned}$$

In the next section we develop a control law based on the IDA-PBC methodology.

3 Stabilization of the Gantry Crane using IDA-PBC Methodology

The basic philosophy of the IDA-PBC methodology is to assign the closed loop dynamics of the system to a desired Hamiltonian system characterized by the triple (M_d, J_2, V_d) standing for the desired inertia matrix, a skew-symmetric matrix and a desired potential energy respectively. We present a brief introduction from [4]. We start with a system whose Hamiltonian is

$$H(q,p) = \frac{1}{2}p^T M^{-1}(q)p + V(q) \tag{10}$$

where $q \in I\!R^n$, $p \in I\!R^n$ are the generalized position and momenta, respectively, $M(q) = M^T(q) > 0$ is the inertia matrix, and $V(q)$ is the potential energy. If we assume that the system has no natural damping, then the equations of motion can be written as

$$\begin{bmatrix} \dot{q} \\ \dot{p} \end{bmatrix} = \begin{bmatrix} 0 & I_n \\ -I_n & 0 \end{bmatrix} \begin{bmatrix} \nabla_q H \\ \nabla_p H \end{bmatrix} + \begin{bmatrix} 0 \\ G(q) \end{bmatrix} u. \tag{11}$$

The matrix $G \in I\!R^{n\times m}$ is determined by the manner in which the control $u \in I\!R^m$ enters into the system and is invertible in the case the system is fully actuated, that is, $m = n$. But for the more difficult case of underactuated system the $\text{rank}(G) = m < n$.

The desired (closed-loop) energy function is assumed to be having the following form:

$$H_d(q,p) = \frac{1}{2}p^T M_d{}^{-1}(q)p + V_d(q) \tag{12}$$

where $M_d = M_d{}^T > 0$ and V_d represent the (to be defined) closed-loop inertia matrix and potential energy function, respectively. We will require that V_d have an isolated minimum at q_* that is

$$q_* = \arg\min V_d(q). \tag{13}$$

In PBC, the control input is naturally decomposed into two terms

$$u = u_{es}(q,p) + u_{di}(q,p) \tag{14}$$

where the first term is designed to achieve the energy shaping and the second one injects the damping. The desired port-controlled Hamiltonian dynamics are taken of the form

$$\begin{bmatrix} \dot{q} \\ \dot{p} \end{bmatrix} = \begin{bmatrix} J_d(q,p) - R_d(q,p) \end{bmatrix} \begin{bmatrix} \nabla_q H_d \\ \nabla_p H_d \end{bmatrix} \tag{15}$$

where the terms

$$\begin{aligned} J_d &= -J_d{}^T = \begin{bmatrix} 0 & M^{-1}M_d \\ -M_d M^{-1} & J_2(q,p) \end{bmatrix} \\ R_d &= R_d{}^T = \begin{bmatrix} 0 & 0 \\ 0 & GK_vG^T \end{bmatrix} \geq 0 \end{aligned}$$

represent the desired interconnection and damping structures, respectively.

The matrix R_d is included to add damping into the system. This is achieved via negative feedback of the (new) passive output $G^T\nabla_p H_d$. Hence second term of (14) can be selected as

$$u_{di} = -K_v G^T \nabla_p H_d \tag{16}$$

where $K_v = K_v{}^T > 0$. The skew-symmetric matrix J_2 (and some of the elements of M_d) can be used as free parameters in order to achieve the kinetic energy shaping.

For the desired closed-loop dynamics, we state the following proposition from [4], which reveals the stabilization properties of IDA-PBC approach.

Proposition 3.1. *The system (15) with (12) and (13) has a stable equilibrium point at $(q_*, 0)$. This equilibrium is asymptotically stable if it is locally detectable from the output $G^T(q)\nabla_p H_d(q,p)$. An estimate of the domain of attraction is given by $\Omega_{\bar{c}}$ where $\Omega_c \triangleq \{(q,p) \in I\!R^{2n} | H_d(q,p) < c\}$ and*

$$\bar{c} \triangleq sup\{c > H_d(q_*,0) | \Omega_c \text{ is bounded}\}. \tag{17}$$

3.1 Energy Shaping

To obtain the energy shaping term, u_{es}, of the controller we replace (14) and (16) in (11) and equate it with (15)

$$\begin{aligned} &\begin{bmatrix} 0 & I_n \\ -I_n & 0 \end{bmatrix} \begin{bmatrix} \nabla_q H \\ \nabla_p H \end{bmatrix} + \begin{bmatrix} 0 \\ G(q) \end{bmatrix} u_{es} \\ &= \begin{bmatrix} 0 & M^{-1}M_d \\ -M_d M^{-1} & J_2(q,p) \end{bmatrix} \begin{bmatrix} \nabla_q H_d \\ \nabla_p H_d \end{bmatrix}. \end{aligned} \tag{18}$$

While the first row of the aforementioned equations is clearly satisfied, the second set of equations can be expressed as

$$Gu_{es} = \nabla_q H - M_d M^{-1} \nabla_q H_d + J_2 {M_d}^{-1} p. \tag{19}$$

Now, it is clear that if G is invertible, that is, if the system is fully actuated, then we may uniquely solve for the control input u_{es} given any H_d and J_2. In the underactuated case, G is not invertible but only full column rank, and u_{es} can only influence the terms in the range space of G. This leads to the following set of constraint equations, which must be satisfied for any choice of u_{es}:

$$G^{\perp}\{\nabla_q H - M_d M^{-1} \nabla_q H_d + J_2 {M_d}^{-1} p\} = 0 \tag{20}$$

where $G^{\perp}$ is a full rank left annihilator of G, that is, $G^{\perp}G = 0$. Equation (20), with H_d given by (12), is a set of nonlinear PDEs with unknowns M_d and V_d, with J_2 a free parameter, and p an independent coordinate. If a solution for this PDE is obtained, the resulting control law u_{es} is given as

$$u_{es} \;=\; (G^T G)^{-1} G^T (\nabla_q H - M_d M^{-1} \nabla_q H_d + J_2 {M_d}^{-1} p). \tag{21}$$

The PDEs (20) can be naturally separated into the terms that depend on p and terms which are independent of p, that is, those corresponding to the kinetic and the potential energies, respectively. Thus, (20) can be equivalently written as

$$\begin{aligned} G^{\perp}\{\nabla_q(p^T M^{-1} p) & \\ -M_d M^{-1} \nabla_q (p^T {M_d}^{-1} p) + J_2 {M_d}^{-1} p\} &= 0 \quad (22) \\ G^{\perp}\{\nabla_q V - M_d M^{-1} \nabla_q V_d\} &= 0. \quad (23) \end{aligned}$$

The first equation is a nonlinear PDE that has to be solved for the unknown elements of the closed-loop inertia matrix M_d. Given M_d, (23) is a simple linear PDE, hence the main difficulty is in the solution of (22).

Clearly, solution to (22) can be simplified if there exists a full rank left annihilator $G^{\perp}$ of G such that

$$G^{\perp}\{\nabla_q (p^T M^{-1} p)\} = 0. \tag{24}$$

This condition essentially imposes that the mass matrix M does not depend on the unactuated coordinate. It is satisfied by many well-known physical examples, for instance, the ball and beam, the VTOL Aircraft and the Acrobot. But in our case, the mass matrix $M(q)$ was found to be dependent on the unactuated coordinate θ, the cable swing angle. To overcome this we employed partial feedback-linearization. Careful observation of (8) reveals that the new inertia matrix is identity and hence satisfying above condition.

3.2 The Matching Equations For Crane Dynamics

Comparing the desired dynamics with actual dynamics after feedback linearization we get,

$$\begin{aligned} &\begin{bmatrix} p_f \\ F_2(q, p_f) \end{bmatrix} + \begin{bmatrix} 0 \\ B_2(q) \end{bmatrix} v_{es} \\ =\; &\begin{bmatrix} 0 & M^{-1} M_d \\ -M_d M^{-1} & J_2(q,p) \end{bmatrix} \begin{bmatrix} \nabla_q H_d \\ \nabla_p H_d \end{bmatrix}. \end{aligned} \tag{25}$$

This gives the following matching equation

$$p_f \;=\; M_d \nabla_{p_f} H_d \tag{26}$$

$$F_2 + B_2 v \;=\; -M_d \nabla_q H_d + J_2 \nabla_{p_f} H_d. \tag{27}$$

Pre-multiplying the above equation by ${B_2}^{\perp}$ (a full rank left annihilator of B_2 such that ${B_2}^{\perp} B_2 = 0$) we have

$${B_2}^{\perp}(F_2 + B_2 v) = {B_2}^{\perp}(-M_d \nabla_q H_d + J_2 \nabla_{p_f} H_d). \tag{28}$$

Writing in coordinate form results in following explicit representation

$${B_2}^{\perp} F_2 = -\frac{g}{q_3} \sin q_1 + {p_f}^T Q(q) p_f$$

where

$$Q = -\frac{1}{q_3} \begin{bmatrix} 0 & 0 & 1 \\ 0 & 0 & 0 \\ 1 & 0 & 0 \end{bmatrix}$$

which gives rise to two PDEs - one for the kinetic energy and the other for the potential energy. Now, solving these two PDEs, (as detailed in [3, 4]), we can design a controller based on the IDA-PBC methodology. Here ${B_2}^{\perp} = \begin{bmatrix} 1 & \frac{\cos q_1}{q_3} & 0 \end{bmatrix}$.

3.3 Solving the Potential Energy PDE

The potential energy PDE can be written using the matching equation as

$$-\frac{g}{q_3} \sin q_1 = -{B_2}^{\perp} M_d \nabla_q V_d. \tag{29}$$

Since mass matrix is identity for the feedback-linearized system, we propose desired mass matrix, M_d, to be a constant matrix of the following form,

$$M_d = \begin{bmatrix} k_1 & k_2 & k_3 \\ k_2 & k_4 & k_5 \\ k_3 & k_5 & k_6 \end{bmatrix} > 0,$$

then the solution of (29) takes the form

$$V_d(q) = -\frac{g \cos q_1}{q_3} + \Phi(z_1(q), z_2(q)) \tag{30}$$

where $z_1(q) = q_3$ and $z_2(q) = (q_2 - \frac{\sin q_1}{q_3})$. Note that the selection of Φ is governed by the condition(13). For this, the necessary condition $\nabla_q V_d(q_*) = 0$ is satisfied if and only if $\nabla\Phi(z_1(q_*), z_2(q_*)) = 0$, while the sufficient condition $\nabla_q^2 V_d(q_*) > 0$ will hold if the Hessian of Φ at the q_* is positive [4]. In our case we choose Φ to be a quadratic function which yields

$$\begin{aligned} V_d(q) \;=\; & -\frac{g \cos q_1}{q_3} + \frac{\alpha_1}{2}(q_3 - q_{3*})^2 \\ & + \frac{\alpha_2}{2}\left(q_2 - q_{2*} - \frac{\sin q_1}{q_3}\right)^2 + \frac{g}{q_3} \end{aligned}$$

where $(0, q_{2*}, q_{3*})$ denotes the equilibrium configuration and $\alpha_i > 0$, $i = 1, 2$ are used as tuning parameters.

3.4 Solving the Kinetic Energy PDE

The KE-PDE is

$$p_f{}^T Q(q) p_f = B_2{}^{\perp}\left[(-\frac{1}{2}M_d \nabla_q p_f{}^T M_d{}^{-1} p_f) + J_2 M_d{}^{-1} p_f\right].$$

Since M_d is chosen to be a constant matrix, it is independent of q. Hence, the KE-PDE gets converted to an algebraic equation as

$$p_f{}^T Q(q) p_f = B_2{}^{\perp} J_2 M_d{}^{-1} p_f \tag{31}$$

Solving this we get J_2 as,

$$J_2 = \begin{bmatrix} 0 & 0 & -2k_6\frac{p_{f1}}{q_3} \\ 0 & 0 & 0 \\ 2k_6\frac{p_{f1}}{q_3} & 0 & 0 \end{bmatrix}.$$

3.5 Energy Shaping Control

The energy-shaping term v_{es} of the control input is synthesized as follows. We have

$$F_2 + B_2 v_{es} = -M_d \nabla_q H_d + J_2 \nabla_{p_f} H_d \tag{32}$$

$$B_2 v_{es} = -F_2 - M_d \nabla_q H_d + J_2 M_d{}^{-1} p_f. \tag{33}$$

Now, it is clear that if B_2 is invertible, that is, if the system is fully actuated, then we may uniquely solve for the control input v_{es} given any H_d and J_2. Since the system is underactuated, B_2 is not invertible but only full column rank, and we have

$$v_{es} = (B_2{}^{\top} B_2)^{-1} B_2{}^{\top}(-F_2 - M_d \nabla_q V_d + J_2 M_d{}^{-1} P_f).$$

Here,

$$M_d = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & k_6 \end{bmatrix}. \tag{34}$$

This gives,

$$v_{es} = \begin{bmatrix} -\frac{(2\dot{q}_3\dot{q}_1 + Z_1\cos q_1 - 2k_6 p_1 p_3)\cos q_1}{q_3{}^2 + \cos q_1{}^2} - \frac{q_3{}^2 Z_1}{q_3{}^2 + \cos q_1{}^2} \\ -k_6\frac{g\cos q_1}{q_3{}^2} + \alpha_1(q_3 - q_{3*}) + \frac{Z_1 \sin q_1}{q_3{}^2} - \frac{g}{q_3{}^2} + \frac{2k_6 p_1{}^2}{q_3} \end{bmatrix}$$

where $Z_1 = \alpha_2(q_2 - q_{2*} - \frac{\sin q_1}{q_3})$. Note that the $k_6 > 0$ is the free parameter which is available for tuning.

3.6 Damping Injection

Damping is achieved via negative feedback of the passive output which is given as:

$$v_{di} = -K_v B_2{}^{\top} \nabla_{p_f} H_d \tag{35}$$

where $K_v = K_v{}^{\top} > 0$ is a damping injection gain. Solving (35) we get

$$v_{di} = -K_v \begin{bmatrix} -\frac{p_{f1}}{q_3}\cos q_1 + p_{f2} \\ p_{f3} \end{bmatrix} \tag{36}$$

Finally, the control input is calculated by combining v_{di} and v_{es} as

$$v = v_{es}(q,p) + v_{di}(q,p). \tag{37}$$

4 Simulations and Results

Simulations were carried out with a twofold objective, first to show that the energy shaping controller proposed with IDA-PBC methodology ensures minimization of cable swing, and second to illustrate the robustness properties of the controller. The damping injection matrix was taken to be of the diagonal form $K_v = \begin{bmatrix} k_v & 0 \\ 0 & k_v \end{bmatrix}$.

Effect of damping injection is illustrated in Fig. 2 for $k_v = 2$ and in Fig. 3 for $k_v = 20$. All other system parameters were kept the same while changing the damping k_v. Mass of the cart, M was taken as 1 kg and that of the payload, m was 0.50 kg. For the same initial conditions the settling time was observed to be increasing with increase in damping injection.

Fig. 4 illustrates robustness property of the controller which is inherent in passivity based control. Here the parameters of the controller are the same as in the case of Fig. 2 while taking M and m to 50 kg and 15 kg, respectively, for the simulations. The system response did not change significantly but the control effort was found to be more.

The initial condition of the swing angle is 10 deg for the results shown in Fig. 2 to Fig. 4. We demonstrate the controller performance with the initial condition of 30 deg in Fig. 5 to emphasize the fact that our controller is effective for even large deviations of θ (outside linear regime). Here all other parameters are kept the same as in the case of Fig.2.

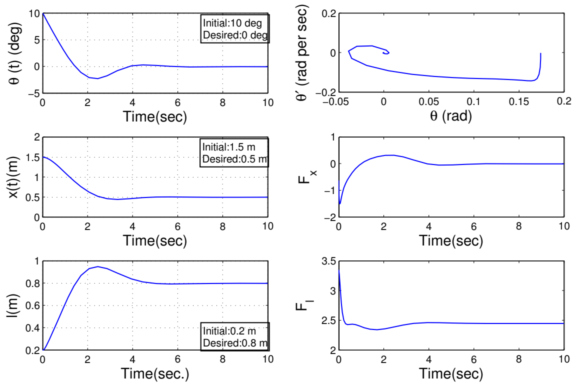

Figure 2: Simulation results for $K_v = 2$.

5 Conclusions

In this paper, we presented an IDA-PBC controller for an overhead gantry crane system. We employed partial feedback-linearization to simplify the solution of PDEs arising from the matching equation. So far, as found in literature the gantry crane problem has been considered with fixed cable length and hence has been modeled as a simple pendulum on a cart for designing control laws. In this paper we considered the cable length as a variable, which closely

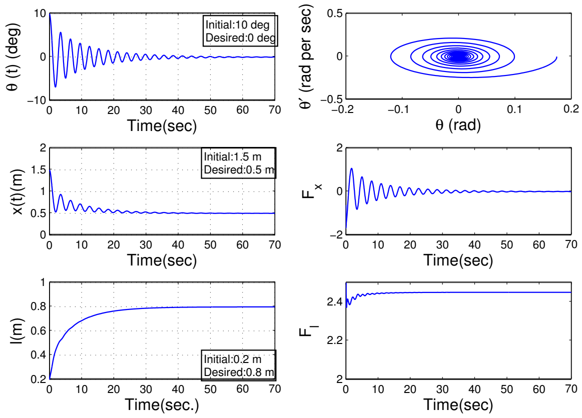

Figure 3: Simulation results for $K_v = 20$.

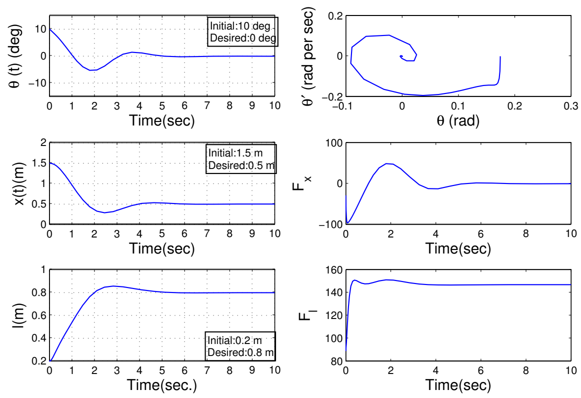

Figure 4: Simulation for Robustness $M = 50$ kg, $m = 15$ kg.

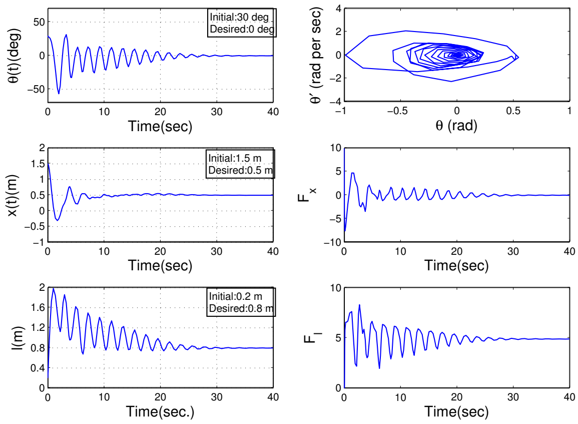

Figure 5: Simulation results for swing angle $\theta = 30$ deg.

replicates real-life crane systems. The control law so developed was found to perform well for the control objective of point to point control with swing minimization as shown in the simulations. The proposed controller was found to be robust for parameter uncertainties like change in mass of the cart as well as the payload.

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