13th National Conference on Mechanisms and Machines (NaCoMM07),
IISc, Bangalore, India, December 12-13, 2007 **NaCoMM-2007-039**

VIBRATION CONTROL USING THREE-LAYER SANDWICH BEAM

B P YADAV
Department of Mechanical Engineering,
Guru Nanak Dev Engineering College, Bidar-585403, India
email: byadavp@yahoo.co.in

Abstract

This paper discusses vibration analysis of three-layer sandwich beam. The present work deals with the analysis of vibration of the primary system having a mass and rubber spring mounted on a three-layer simply supported symmetrically arranged with elastic faces and viscoelastic core sandwich beam. Based on the derivation of R.C. Das Vikal et. al, for response of harmonically excited mass and for transmissibility, the effectiveness of three-layered sandwich beam of viscoelastic core for central mounting of the primary system is evaluated. The thickness of viscoelastic core is varied to get minimum value of response and transmissibility.

Keywords: Sandwich beam, Response, Transmissibility

1. Introduction

There has been a constant need for the light weight and high strength materials for various applications like aerospace and automobiles. The sandwich structures are relatively lighter in weight and less expensive. In sandwich structures the second moment of area of the cross-section of the structure is increased by separating face-sheets with a low density material, known as core of the sandwich structure. The face-sheets are made of a high strength and high modulus material. This results in the increased stiffness and load bearing capacity of the sandwich structure in bending.

In this paper the problem of a primary vibration excitation system in contact with a sandwich beam is considered. Work on the analysis of flexural vibrations of sandwich beams has been reported by many investigators [1-6]. To simplify the analysis these investigators have taken into consideration only the strain energy due to bending and longitudinal deformation of the elastic faces and that due to shear deformation of the core. This assumption is justified on practical grounds if the Young's modulus of the core is much smaller than the Young's modulus of the outer layers. In the work reported here, both the vibration response of a flexibly supported mass attached to a viscoelastic core sandwich beam at its centre and the force transmissibility provided by the complete system have been computed, as both these aspects are important from the point of view of vibration control.

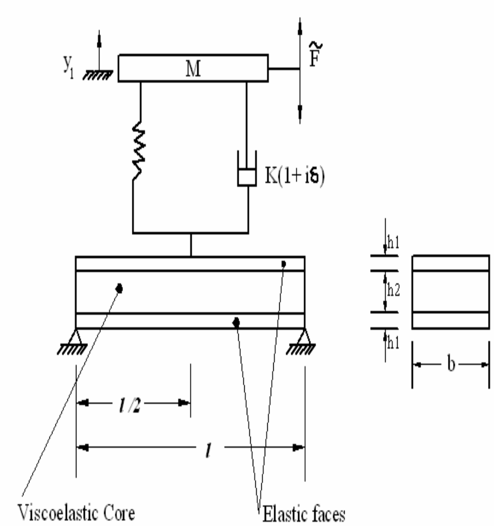

Figure 1. Mathematical model and beam geometry

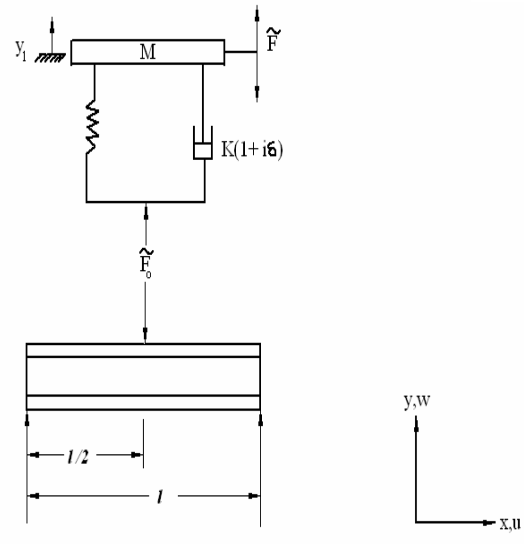

Figure 2. Free body diagram

2. Problem Formulation

Figure 1 shows the model system, which consists of a flexibly supported excitation system attached to the centre of a simply supported sandwich beam having a viscoelastic core and elastic faces. The vibrating excitation system, here designated as the primary system, consists of a mass and rubber spring whose dynamic charateristic is defined by the equation K* = K (1+ iδ), where K and δ are the stiffness and the loss factor respectively. A three-layered sandwich beam has face layers of thickness h_1 and a core thickness h_2. The face layers are purely elastic with Young's modulus E. The core has a shear modulus G(1+iβ), β being the loss factor of the core material. Figure 2 shows the free body diagrams of the primary system and the sandwich beam. The concentrated harmonic force $\bar{F}$o is acting on the beam. The sandwich beam is analyzed for its dynamic stiffness with respect to Fo.

3. Method of solution

The differential equation of a sandwich beam having elastic faces and viscoelastic core is [6]

$$\partial^6 w/\partial x^6 - g(1+Y)\,\partial^4 w/\partial x^4 = (1/D_t)(\partial^2 p/\partial x^2 - gp). \quad (1)$$

The assumptions under which equation (1) has been derived are mentioned in reference [6]. For harmonic motion one can assume that

$$w(x,t)=W(x)\exp(ift). \quad (2)$$

Then the inertia force p has the form

$$p=-m\partial^2 w(x,t)/\partial t^2=Wf^2\exp(ift) \quad (3)$$

Substitution of p and w from equations (2) and (3) into equation (1) gives

$$d^6W/dx^6 - g(1+Y)\,d^4W/dx^4 - (mf^2/D_t)d^2W/dx^2+(mf^2/D_t)gW=0, \quad (4)$$

which is a simple linear differential equation of sixth order. Hence a solution of the form

$$W(x)=A\exp(\sigma x) \quad (5)$$

can be assumed. Substitution of equation (5) into equation (4) yields the characteristic equation

$$\sigma^6-g(1+Y)\sigma^4-(mf^2/D_t)\sigma^2+(mf^2/D_t)g=0, \quad (6)$$

which is cubic in σ^2. The roots can be exactly determined [10]. The complete solution of differential equation (4) can then be expressed as

$$W(x)=\sum_{j=1}^{6} A_j\exp(\sigma_j x). \quad (7)$$

The constants A_j, j = 1, 2, ..., 6, are to be obtained by application of the boundary conditions of the beam.

3.1. Boundary Coditions

The beam can be imagined to be comprised of identical halves, each of which is acted upon by one-half of the applied force Fo at the junction point. The centre of the beam can now conveniently be taken as the origin.

For writing the boundary conditions one needs the expressions for P_1, P_3, u_1, u_3,
and so on. These can be easily derived [6] and expressed as follows:

$$P_1=-P_3=(D_t/gd)[d^4W/dx^4-gY\,d^2W/dx^2-(mf^2/D_t)W], \quad (8)$$

$$u_1=(D_t/g^2 E h_1 db)[d^5W/dx^5-gY\,d^3W/dx^3-\{(mf^2/D_t)+g^2Y\}dW/dx], \quad (9)$$

$$\text{Bending moment}=(D_t/g)[-d^4W/dx^4+g(1+Y)d^2W/dx^2+(mf^2/D_t)W, \quad (10)$$

$$\text{Shear force}=(D_t/g)[-d^5W/dx^5+g(1+Y)d^3W/dx^3+(mf^2/D_t)dW/dx], \quad (11)$$

The possible boundary conditions for a sandwich beam free at one end and simply supported at the other end are as follows:

at x = 0 (at centre)
(i) shear force = Fo/2, (ii) slope = dW/dx = 0,
(iii) u_1 = 0;

at x = l/2 (at right end)
(iv) defection = W = 0, (v) bending moment = 0,
(vi) $P_1 = -P_3 = 0$.

Applying the above six boundary conditions, with the help of equations (8) – (11), one obtains finally a matrix equation of the form

$$[C]\{B\}=\{H\}, \quad (12)$$

where [C] is a square matrix of dimension 6x6. {B} and {H} are column matrices. The elements of these matrices are, for j = 1, 2, ..., 6,

$C_{1j} = -\sigma_j^5 + g(1+Y)\sigma_j^3$, $C_{2j} = \sigma_j$, $C_{3j} = \sigma_j^5 - gY\sigma_j^3$,
$C_{4j} = \exp(\sigma_j l/2)$, $C_{5j}=\sigma_j^2\exp(\sigma_j l/2)$, $C_{6j}= \sigma_j^4\exp(\sigma_j l/2)$,
$B_j = A_j/Fo$, $H_j = \frac{g}{2D_t}$, j = 1,
= 0, j ≠ 1.

Equation (12) can be solved for B_1, B_2, ..., B_6. The beam solution then can be written as

$$\frac{W(x)}{Fo}=\sum_{j=1}^{6} B_j\exp(\sigma_j x). \quad (13)$$

4. Response of Primary System and Transmissibility

The equations of Response of Primary System and Transmissibility of a sandwich beam having elastic faces and viscoelastic core are [7]

$$Y_1 / F = \left\{\sum_{j=1}^{6} B_j + \frac{1}{K(1+i\delta)}\right\} / \left\{-Mf^2 \sum_{j=1}^{6} B_j - \frac{Mf^2}{K(1+i\delta)} + 1\right\}, \quad (14)$$

$$T = \frac{2D_t}{g} \frac{\sum_{j=1}^{6}\left[\left\{-\sigma_j^5 + g(1+Y)\sigma_j^3 + \left(\frac{mf^2}{D_t}\right)\sigma_j\right\} B_j \exp\left(\frac{\sigma_j l}{2}\right)\right]}{\left[-Mf^2 \sum_{j=1}^{6} B_j - \frac{Mf^2}{K(1+i\delta)} + 1\right]} \quad (15)$$

Table 1: Response of Primary System and Transmissibility at Constant Frequency and Varying Core Thickness.

Sl No.	Frequency (cps)	Thickness of core (h_2) (cm)	$\frac{Y_1}{F}$ (cm/N)	T (db)
1.	10	0.2	0.0067	5.72
2.	10	0.4	0.0021	14.58
3.	10	0.6	0.0017	18.03
4.	10	0.8	0.0024	16.44
5.	10	1.0	0.0012	38.42

5.Results and Discussion

Theoretical results deduced from equations (14) and (15) are plotted in Figures 3 and 4. Figures 3 and 4 show respectively plots of theoretical curves representing the variation of the response of the primary system and the variation of the transmissibility provided by the complete system with core thickness.

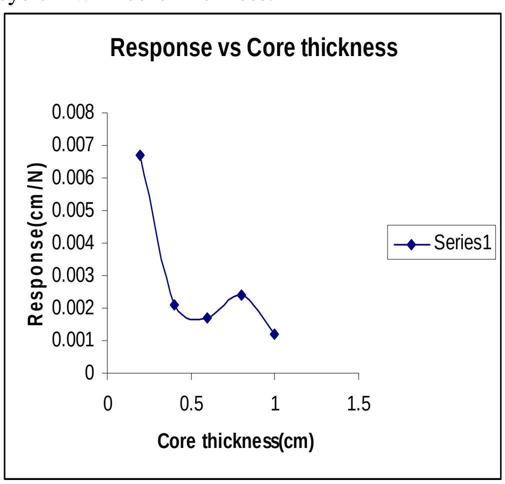

Figure 3 Response vs Core thickness at constant frequency (10cps)

The beam used is a MS – PVC – MS sandwich beam. Its dimensions were h_1= 1 mm, h_2 is varied from 2 mm to 10 mm, l =500 mm and b = 80 mm. The dynamic properties of rubber and PVC were taken from experimentally obtained values [8, 9] and these are given in Appendix 1. The main mass was taken as 1.8 kg. It can be seen that the increase of core thickness results in a decrease of response but increase of transmissibility at the constant frequency of 10 cps. Hence suitable values of core thickness is taken where both response and transmissibility are less.

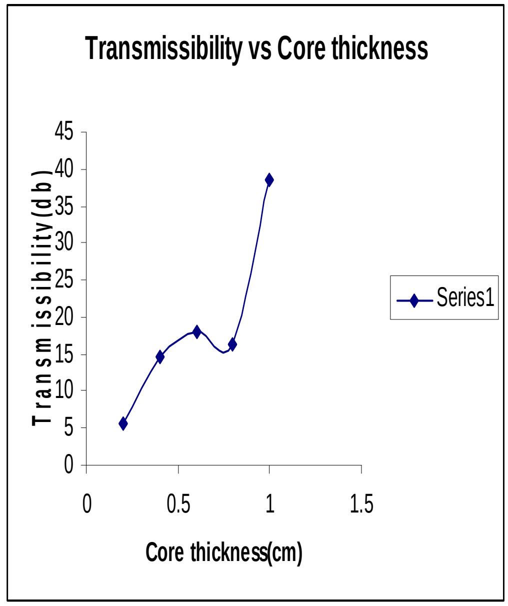

Figure 4 Transmissibility vs Core thickness at constant frequency (10cps)

6. Conclusion

It can be concluded that a three-layered sandwich beam having a configuration symmetrical with respect to both geometrical and physical parameters provides a minimum response to the primary system as well as minimum transmissibility of the excitation force to the support. Further, it is found that the increase of core thickness h_2 results in a decrease of response but increase of transmissibility at the constant frequency of 10 cps. Hence suitable value of core thickness h_2 is taken where both response and transmissibility are less.

References

1.P. Grootenhuis and D. J. Ewins, “Multidirectional Vibration Isolation”, Presented at the Vibration Isolation Symposium of the Society of Environmental Engineers held at Imperial College, London, 1972.

2. D. J. Mead, “The Double Skin Damping Configuration,” University of Southampton A. A. S. U. Report No. 160, 1962.

3. R. A. DiTaranto, “Theory of Vibratory Bending for Elastic and Viscoelastic Layered Finite-length Beams,” Journal of Applied Mechanics 32, 881 – 886, 1965.

4. Y. Y. Yu, “Damping of Flexural Vibrations of Sandwich Plates.”Journal of Aerospace Sciences 29, 790 – 803, 1962.

5. D. Ross, E. E. Ungar and E. M. Kerwin, “Damping of Flexural Vibrations by Means of Viscoelastic Laminae.” Colloquium on Structural Damping, ASME, 1959.

6. D. J. Mead and S. Marcus, “The Forced Vibration of a Three – layer, Damped Sandwich Beam with Arbitrary Boundary Conditions.” Journal of Sound and Vibration 10, 163 – 165, 1969.

7. R. C. Das Vikal, K. N. Gupta and B. C. Nakra, “Vibration of an Excitation System Supported Flexibly on a Viscoelastic Sandwich Beam at its Mid-Point.” Journal of Sound and Vibration (1981) 75 (1), 87 – 89.

8. R. C. Vikal, “Two Stage Isolation System Using Viscoelastic Materials.” M. Tech. Thesis, 1976, Indian Institute of Technology, Delhi.

9. N. T. Asnani, “Analysis of Multilayer Beams with Constrained Viscoelastic Layer.” Ph. D. Thesis, Indian Institute of Technology, Delhi.

10. C. J. Tranter, “Techniques of Mathematical Analysis.” London: The English University Press Limited, 1957, see pp. 131-132.

Appendix 1: Dynamic Properties of Rubber and PVC Materials

Dynamic stiffness and loss factor of rubber [8] at 30^0C: K = 850.0 + 28.3 f_c N/cm, for f_c ≤ 45 Hz; K = 2500 N/cm, for f_c > 45 Hz; δ = 0.126.
Shear modulus and loss factor of PVC [9] at 30^0C: G = 420.0 + 2.5 f_c N/ cm^2 and β = 0.24 + 0.00125 f_c for f_c ≤ 80 Hz; G = 570.0 + 0.667 f_c N/ cm^2 and β = 0.28 + 0.00075 f_c for f_c > 80 Hz.

Appendix2: Nomenclature

B_j = A_j/ F_0, j = 1,2, …, 6
b width of beam
d distance between neutral axes of elastic layers, = h_1+h_2
D_t overall bending stiffness of elastic layers about their neutral axes, = Eh_1^3/ 6 b
E Young's modulus of elastic layers
F exciting force
f frequency in rad/s, and f_c frequency in Hz
G in-phase shear modulus of core material
G^*complex shear modulus of core material, = G (1+ iβ)
g shear parameter, = 2 G^*/ E $h_1 h_2$
h_j thickness of jth layer
i $\sqrt{-1}$
K dynamic stiffness of rubber material
l length of beam
m mass of the beam per unit length
M vibrating mass
T transmissibility
x co-ordinate along the length of beam
Y_1 displacement of vibrating mass, function of x
y_1 displacement of vibrating mass, function of x and t
Y dimensionless geometrical parameter, = (d^2 b/ D_t) (Eh_1/2)
β loss factor of core material
δ loss factor of rubber
σ_j complex roots of characteristic equation (6)