

Structural synthesis of kinematic chains of lever mechanisms

Doc. N. Krokmal, CSc

Department of Machine details /Faculty of Transport systems/KSU, Kurgan, Russia

Email: KrokNN@yandex.ru

Abstract

In this paper structural analysis and synthesis of kinematic chains of lever mechanisms and structure groups (which are known as Assur groups) are submitted. These analysis and synthesis base on the new opened structural properties of the kinematic chains. Conditions and limitations, which were accepted, discussed.

Keywords: Kinematic Chains, Analysis, Synthesis

1 Introduction

One of the tasks in the theory of mechanisms is researching principles of building structural schemes of lever mechanisms, including special types of kinematic chains - structural groups.

For a solution of this problem various techniques were offered. [1-4]. The traditional approach was the selection of feasible mechanisms based on some observations. Infeasible and isomorphic representatives were deleted from the search according to the set of constraints. Though many of these techniques were successfully used, it is obvious, that they become ineffective at a solution of complex problems. It is seems expedient for their solution to use the techniques based on the mathematical graph theory and a theory of combinations. So, for classification of a structure of mechanisms Freudenstein used the theory of Polya [5]. He together with Buchsbaum [6] used network concepts and the combinatory analysis for synthesis of kinematic chains of mechanisms. Yan and Hwang [7] proposed the algorithm based on concepts of the combinatory theory and permutation groups, for count the number of non-isomorphic mechanisms with the required number and types of links and joints of kinematic chains. Yan and Hung [8] presented a method of identification and count the number of non-isomorphic mechanisms from kinematic chains subject to constraints of adjacency or incidence relations among links and joints, based on the theory of Polya and a generating function. They expanded their previous work and presented concept of the modified permutation groups [9]. Using variables to indicate the remaining links in modified permutation groups and based on concept of a generating function and the theory of Polya it is possible to work out more constraints simultaneously. Yan and Hung [10] have presented also a technique for count-

ing the number of non-isomorphic mechanisms with required constraints based on generating function and the Theory of Polya.

However, existing methods have a some shortages. The basic among them is lack of uniformity of construction of schemes. It can be overcome by studying of their general structural properties.

Among other known principles of structural classification of lever mechanisms, the Assur's principle is fruitful. The main structural element in this classification of mechanisms is the structural group (further SG) – Assur group. Classical definition of structural group is formulating as follows «... indivisible kinematic chain which degree of mobility is equal to zero at connection in its external kinematic pairs to a frame». However, what means an attribute «indivisible kinematic chain»? More often, this understands as that the given kinematic chain cannot divide into more simple kinematic chains. But following examples among many others show, that kinematic chains, being structural groups, in practice it is possible to build of more simple kinematic chains as shown in Fig.(1).

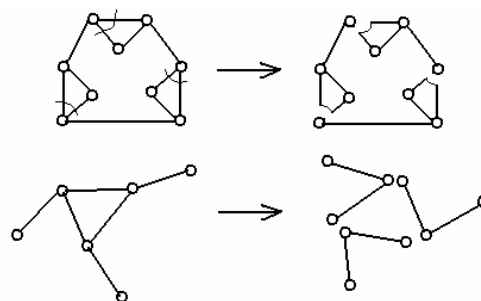


Figure 1: The dividing of SG on elementary chains.

Nevertheless resulted, these schemes are schemes of structural groups. From here, it is possible to draw a conclusion that the attribute - «indivisible» in definition of structural groups is necessary for specifying research of their structure more in detailed.

2 Basic Assumptions and Parities Accepted at Structural Analysis of Kinematic Chains

Let's consider flat structural groups, which parts are connecting among them by, revolute kinematic pairs - flat hinges. As shown in [11] such assumption will not influ-

ence a generality of reasoning, but will allow presenting SG by corresponding mathematical objects - simple graphs. Generally SG contains parts with two kinematic pairs - levers, and parts which have more than two hinges - base parts Fig. (2).

For unification of SG structure, we shall present every base part as a truss consisting of levers, which are connecting by hinges so that the base part has been broken into rigid triangles as shown in Fig. (2).

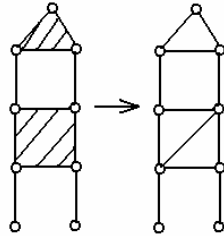


Figure 2: Representation of base part as a truss.

Such replacement does not break parity between number of parts and number of kinematic pairs that Tchebyshev's formula (for definition a degree of SG mobility) is fair. Really, let the base part has m points in which hinges are located and which are motionless from each other. At replacement of a base part by a truss, these points also should remain motionless from each other. Therefore, in a flat truss between them it is necessary to install $2m-3$ connections (levers). Thus, m levers it is possible to place so, that they form an external contour of a base part and $2m-3-m = m-3$ levers form connections between tops inside a contour. Then it is easy to count up, that the number of hinges in a truss is equal $3m-6$. (At replacement of a base part by a truss the number of base parts will decrease on one, but the number of levers and hinges will increase). Therefore, after similar replacement, Tchebyshev's formula can be writing down:

$$3 \cdot (n - 1 + (2 \cdot m - 3)) - 2 \cdot (p + (3 \cdot m - 6)) = 3 \cdot n - 2 \cdot p \quad (1)$$

Here p is number of hinges in initial SG, n is number of parts in initial SG.

Thus, at replacement of base parts with trusses it is received equivalent mechanical system.

3 Structural Analysis of SG

Let's correspond structural group to its graphic representation - graph. In the graph, edges are images of levers, and tops - the points where kinematic pairs are displaying.

Any internal top of the graph has a degree (number of incidental edges) $\alpha \geq 3$ and corresponds to $\alpha - 1$ kinematic pairs. Each external top of the second degree corresponds to two hinges. The external top of the first degree corresponds to one hinge. Therefore, Tchebyshev's formula for SG graph can be writing down as follows:

$$\frac{3 \cdot n}{2} = p_1 + 2 \cdot p_2 + \sum_{\alpha=3} (\alpha - 1) \cdot p_\alpha \quad (2)$$

Here p_1 is number of tops of the first degree, p_2 is number of tops of the second degree, p_α is number of tops of the α degree.

Based on the known parity of the graph's theory [12] the formula is fair for SG graph:

$$2 \cdot n = p_1 + 2 \cdot p_2 + \sum_{\alpha=3} \alpha \cdot p_\alpha \quad (3)$$

Subtracting Eq. (2) from Eq. (3), we shall receive parity between number of edges and number of internal tops for SG graph:

$$\frac{n}{2} = \sum_{\alpha=3} p_\alpha \quad (4)$$

Let's introduce for SG graph operation of splitting of a top, which consists of two steps:

- 1) A degree of the top reduces by elimination of an incidental edge.
- 2) A new top place on the eliminated edge as shown in Fig. (3).

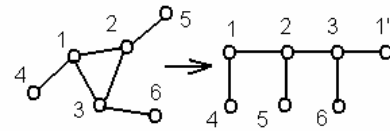


Figure 3: Operation of splitting of the graph's top number 1.

Graph of any structural group can be presenting in the form of a dichotomizing tree by the help of operation of splitting as shown in Fig. (3) and in Fig. (4).

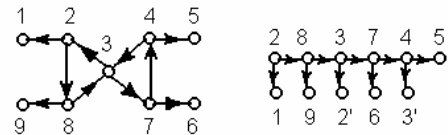


Figure 4: Directed SG graph and its dichotomizing chain.

Let's pass from the graph to the directed SG graph as shown in Fig. (4). In this case a necessary and sufficient condition of construction of a dichotomizing tree is existence only two arcs leaving each internal top of the graph. Since applying operation of splitting to each top and entering arcs (keeping only one input arc in a top), we shall receive a dichotomizing tree Fig. (4). In this case, external tops of the first and the second degrees must have only input arcs.

For any directed graph, it is possible to build a 0, 1 adjacency matrix [12] in which number of every line and a column corresponds to number of a top in the graph. The element of a matrix is equal 1 if the arc input into the top with number of a column from the top with number of a line. The element of a matrix is equal 0 in other case. For such matrix the sum of elements in lines is equal to the sum of elements in columns (number of output arcs is equal to number of input arcs), therefore for any directed SG graph it is possible to build following Table 1.

Table 1: Distribution of entering and coming arcs in SG graph

The degree of a top	The common number of outputs	The common number of inputs
1	-	p_1
2	-	$2 \cdot p_2$
α	$2 \cdot p_\alpha$	$(\alpha - 2) \cdot p_\alpha$

Based on the noted property of a adjacency matrix we shall write:

$$\sum_{\alpha=3} 2 \cdot p_\alpha = p_1 + 2 \cdot p_2 + \sum_{\alpha} (\alpha - 2) \cdot p_\alpha \quad (5)$$

Or

$$p_1 + 2 \cdot p_2 + \sum_{\alpha=3} \alpha \cdot p_\alpha - \sum_{\alpha=3} 4 \cdot p_\alpha = 0 \quad (6)$$

Eq. (4) received above, therefore after its substitution in Eq. (6) we shall have:

$$p_1 + 2 \cdot p_2 + \sum_{\alpha=3} \alpha \cdot p_\alpha - 2 \cdot n = 0 \quad (7)$$

We come to know Eq. (3), as confirms an opportunity of realization of the set directions of arcs in directed SG graph. I.e. graph of any structural group can be presenting in the form of a dichotomizing tree by the help of operation of splitting.

Numerous examples of decomposition of SG allow assuming that any SG can be spreading out not only in a dichotomizing tree, but also in a dichotomizing chain as shown in Fig. (5). It is possible to assume also, that mathematically it corresponds to finding Hamiltonian path between internal tops for the SG graph. It is obvious that the dichotomizing chain can be constructing for the some SG connected with each other consistently. If any SG connected in parallel, such connection can be spreading out in a dichotomizing tree. Search Hamilton's way into the graph carry out according to the algorithm [13] for example.

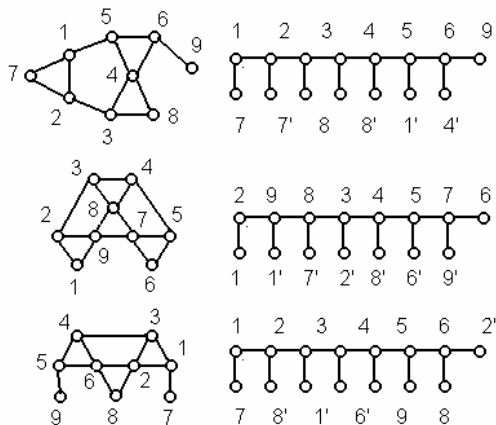


Figure 5: Examples of decomposition of SG graphs in dichotomizing chains.

The dichotomizing tree and dichotomizing chain are represented in Fig. (6). As against a dichotomizing tree the dichotomizing chain has no bifurcations and its any top is adjacent to top of the first degree.

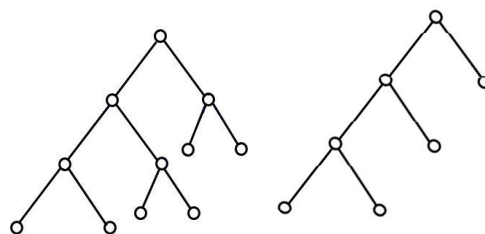


Figure 6: Dichotomizing tree and dichotomizing chain.

4 Structural Synthesis of SG

If any SG graph can be presenting in the form of a dichotomizing chain by the operation of splitting of tops, it is obvious, that it is possible to receive SG graph or graph of consistently connected groups from a similar chain by return operations.

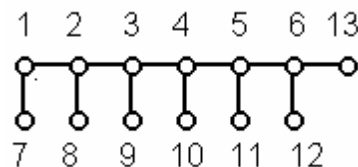


Figure 7: Initial graph.

Initial graph can consist of any number of consistently connected dyads as shown in Fig. (7). In this case, operation of synthesis of SG graph is reducing to overlapping trailing tops of the initial graph with its internal tops, i.e. operation of overlapping of tops is return to operation of splitting of tops.

It is always possible to correspond the initial graph to the initial adjacency matrix Table 2. Such matrix contains $(2 \cdot d + 1)$ columns and d lines, here d is number of dyads in the initial graph. In the initial graph, internal tops numbered at first and then trailing tops are numbered. Therefore, the matrix consists of two parts. In the left part, columns with numbers of internal tops, and in the right part columns with numbers of trailing tops are located. The letter «e» in a matrix means, that the internal top corresponding number of a line is connecting by an edge with the internal top corresponding number of a column. Figure «1» in the right part of an initial matrix means that the top corresponding number of a line is connecting by the edge with a trailing top corresponding number of a column. The letter «i» means the same, as figure «1», but using for convenience of the matrix description.

Table 2: Adjacency matrix of the initial graph

№	1	2	3	4	5	6	7	8	9	10	11	12	13
1	<i>x</i>	<i>e</i>					1	<i>x</i>	<i>x</i>	<i>x</i>	<i>x</i>	<i>x</i>	<i>x</i>
2	<i>x</i>	<i>x</i>	<i>e</i>					1	<i>x</i>	<i>x</i>	<i>x</i>	<i>x</i>	<i>x</i>
3		<i>x</i>	<i>x</i>	<i>e</i>					1	<i>x</i>	<i>x</i>	<i>x</i>	<i>x</i>
4			<i>x</i>	<i>x</i>	<i>e</i>					1	<i>x</i>	<i>x</i>	<i>x</i>
5				<i>x</i>	<i>x</i>	<i>e</i>					1	<i>x</i>	<i>x</i>
6					<i>x</i>	<i>x</i>						1	<i>i</i>

Operation of overlapping of the tops into the initial graph leads to synthesis of SG or generally the graph of a kinematic chain. This is correspond to moving «1» along a line from the right part of the initial matrix to any free cell of the left part. In the SG graph there cannot be loops and the double edges, therefore a symbol «*x*» notes corresponding cells. Thus, synthesis of SG graphs is reducing to the search of variants of accommodation «1» in the left part of a matrix, by their moving from the right part.

Moving «1» in the area of the right part of a matrix corresponds to the formation of external top of the second degree and this is a special case. However, it is interesting to note, that overlapping of two trailing tops, incidental to the same edge gives a triangular part with external top Fig. (2). And overlapping of trailing tops, incidental to different edges, leads to formation «paradoxical» groups [14]. Overlapping three or greater number of trailing tops in the initial graph is essentially possible in special case.

It was marked above that from the initial graph it is possible to receive both SG graphs, and graphs of more complex compound kinematic chains. It is meaningful to synthesize structural groups separately. Obviously, thus on moving «1» in the initial matrix some restrictions are imposed. Let's consider these restrictions more in detail.

Firstly, in the right part of initial matrix should remain not less than four «1». Really, if in SG graph to reject all external tops with incidental edges it will turn out graph of the closed kinematic chain which degree of relative mobility should be ≥ 1 . Differently it will be not a kinematic chain, but a base part. There is only one such group Fig. (3), since in this case the condition of static definability (for the group) will be:

$$w = 3 \cdot n - 2 \cdot (n - 1) \cdot 2 = -n + 4 = 0 \quad (8)$$

Here w is a degree of mobility of a kinematic chain, n is number of parts in the group. Whence $n=4$

If degree of relative mobility of an internal kinematic chain is more or equal 1 it is possible to write down a following inequality:

$$w = 3 \cdot (n - 1) - 2 \cdot p \geq 1 \quad (9)$$

Where n is number of edges in the graph, p is number of kinematic pairs.

The number of edges in the graph after rejection of external tops is defining by a following parity:

$$n = 2 \cdot d - s \quad (10)$$

Here d is number of dyads in the initial graph, s is number rejected levers (number of «1» in the right part of a adjacency matrix).

The number of kinematic pairs in the remained kinematic chain is giving by expression

$$p = 2 \cdot (d - 1) + 1 + (d + 1) - 2 \cdot s = 3 \cdot d - 2 \cdot s \quad (11)$$

Let's substitute Eq. (10) and Eq. (11) in Eq. (9), after transformations we shall have:

$$w = s - 3 \geq 1 \text{ Or } s \geq 4$$

It gives minimally possible number of «1» which should be remaining in the right part of an initial matrix.

Secondly, SG should be «indivisible» kinematic chain. Definition of the «indivisible» kinematic chain can be giving, proceeding from the following reasoning. If SG is «indivisible» at its formation trailing tops of the bottom level the initial graph necessarily join its internal tops so that coordinates of cells (n, m) and (i, j) of any two nearest lines in the left part of initial matrix in which settle down «1» satisfied to the parities:

$$n > i, i \geq m, j < m.$$

Thus, the feedbacks between dyads in SG are carrying out as shown in Fig. (8) and in Table 3.

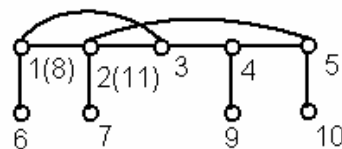


Figure 8: Feedbacks in the graph of the structural group.

Table 3: Adjacency matrix of SG graph Fig. 7

№	1	2	3	4	5	6	7	8	9	10	11
1	<i>x</i>	<i>e</i>				1	<i>x</i>	<i>x</i>	<i>x</i>	<i>x</i>	<i>x</i>
2	<i>x</i>	<i>x</i>	<i>e</i>				1	<i>x</i>	<i>x</i>	<i>x</i>	<i>x</i>
3	1	<i>x</i>	<i>x</i>	<i>e</i>					<i>x</i>	<i>x</i>	<i>x</i>
4			<i>x</i>	<i>x</i>	<i>e</i>				1	<i>x</i>	<i>x</i>
5				<i>x</i>	<i>x</i>						1

In the graph of the structural group all internal tops are placed feedback. This is the attribute of a group corresponding to the term understanding as «indivisible kinematic chain».

Presence of feedbacks in SG, overlapping all tops in its graph, corresponds to the fact that the system of the equations describing connections between tops in it does not break up to subsystems, which can be solving consistently one after another.

In a mechanism, feedbacks are absent between structural groups, therefore it is possible to spend the analysis of the mechanism consistently on groups, solving systems of the equations for each group separately.

If two or more structural groups to connect among them so that feedbacks in the new graph were blocking, the new structural group is forming in this case Fig. (9).

Thirdly, in SG graph there cannot be double arcs. Their presence would mean that there are duplicating parts in a kinematic chain. Therefore, should not be «1» located

symmetrically concerning the main diagonal of the left part of SG matrix.

The algorithm of search of variants of accommodation «1» is realized in the computer program with an opportunity to spend synthesis as structural groups and compound kinematic lever chains.

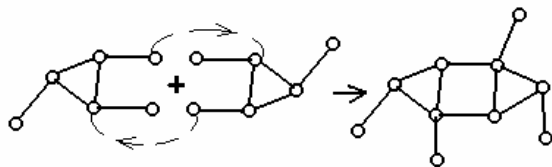


Figure 9: Formation new SG from two SG groups.

At synthesis of the structural groups, it turns out many isomorphic graphs. It is required to spend selection none isomorphic representatives. It is a typical problem of the graph's theory in the mathematics. There are enough algorithms for the decision such problem, for example [15]. However, at search of the big number of variants the algorithm, giving high speed of calculations with reference to conditions of a problem is necessary.

At SG synthesis by the considered way a problem arise which essence can understand from Fig. (10). The structural group presented by the graph (a) is identical to the group presented by the graph (b). However, graphs (a) and (b) are not isomorphic. Graph (a) have specific internal top number 6 in which there is no relative mobility of edges incidental to it. In practice, such top can serve for formation in it the kinematic pair with other structural group. The similar top can be formed on any part by connection to it a dyad. Let's name the tops similar to the top 6 as a passive top as shown in Fig. (10). At performance of SG synthesis, it is meaningful to reject the graphs containing such tops.

The distinctive property of passive tops noted above, allows building the algorithm for their revealing. This algorithm providing following operations with an adjacency matrix:

- 1) Choose a next edge of the graph, not incidental to a trailing top.
- 2) Find tops, adjacent with both tops, incidental to the given edge of the graph. Thus, we install set tops and accordingly the edges belonging to the base part.
- 3) Step 2 carries out for each edge of the base part before full exhaustion of tops.

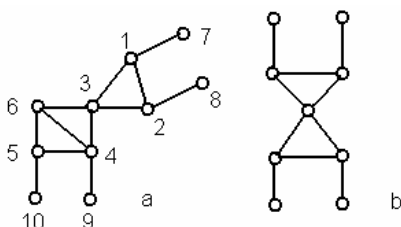


Figure 10: Graphs of identical structural groups.

- 4) Check up each top of a base part, whether it has the incidental top which is not belonging a base part.

- 5) If the top of the base part, which is not having incidental tops outside of this part, found out then it is passive and graph can be rejecting.

There are five kinds of the flat dyads Fig. (11).

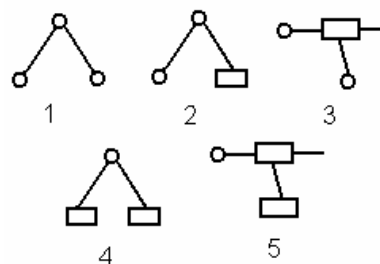


Figure 11: Kinds of the flat dyads.

Therefore the classical problem of combination theory for everyone synthesized graph SG can be put. This is problem of definition of variants of an arrangement of dyads of each kind in a dichotomizing chain of the graph. Such problem can be solving in conformity, for example, by the help of algorithm considered in [13].

5 Formation of Spatial SG

At formation of spatial SG as the entry condition of static definability, we shall accept the condition:

$$6 \cdot n - 4 \cdot p_4 = 0 \quad (12)$$

I.e. preliminary we shall build kinematic chains containing only spatial kinematic pairs of the 4-th class Fig. (12). The similar technique is using in [16].



Figure 12: Kinematic pairs of the fourth class.

The parity of the static, which written down above, is equivalent to the following characteristic equation for flat SG:

$$3 \cdot n - 2 \cdot p_4 = 0 \quad (13)$$

Therefore, everything that stated for the flat groups it is fair for the spatial groups having pairs of the 4-th class only. Hence, such groups represent system of consistently connected dyads with feedbacks. If a group with pairs of the 4-th class is in disposal, it is possible to receive a group containing pairs of the 3-rd and the 5-th class. There are two known ways for this purpose:

- 1) Replacement two pairs of the 4-th class by one pair of the 3-rd and one pair of the 5-th class, thus the balance of degrees of freedom of a dyad is keeping Fig. (13). Number of variants of an arrangement of kinematic pairs in a dyad at such way of formation of spatial group can be only four: 4 - 4 - 4; 4 - 5 - 3; 5 - 3 - 4; 5 - 4 - 3.
- 2) A pair of the 4-th class replace by two pairs of the 5-th class connected by lever.

At performance of kinematical analysis of any spatial SG, it is possible to solve a return problem - reductions the

graph of this group to the scheme containing only pairs of the 4-th class. It will allow spreading out a graph of a group in a dichotomizing chain to reveal a structure of feedbacks between dyads in the group.

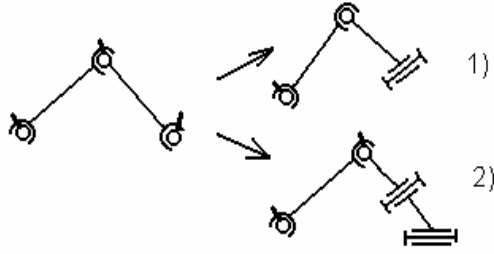


Figure 13: Transformation of the spatial dyad.

6 Transfer Function of SG

As shown above any SG consists of consistently connected dyads with feedbacks. Every dyad is the elementary transforming device with two entrances (external kinematic pairs) and one exit (internal kinematic pair). Moving or speeds give to entrances with ones parameters and they take from exit with transformed parameters. Hence, each dyad has transfer function. There are five kinds of flat dyads and some tens of spatial dyads. For each kind, transfer function is available. Let's show how to define these transfer function for the first kind of dyads Fig. (14).

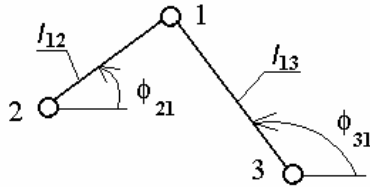


Figure 14: Scheme of the dyad for definition its transfer function.

For this dyad, we shall write down the system of equations:

$$\left. \begin{aligned} (x_1 - x_2)^2 + (y_1 - y_2)^2 &= l_{12}^2 \\ (x_1 - x_3)^2 + (y_1 - y_2)^2 &= l_{13}^2 \end{aligned} \right\} \quad (14)$$

Here x_i, y_i are coordinates of the appropriate points in chosen Cartesian system of coordinates; l_{ij} are lengths of the appropriate levers.

After differentiation Eq. (14), we receive the following system of equations:

$$\left. \begin{aligned} (x_1 - x_2) \cdot (x_1' - x_2') + (y_1 - y_2) \cdot (y_1' - y_2') &= 0 \\ (x_1 - x_3) \cdot (x_1' - x_3') + (y_1 - y_3) \cdot (y_1' - y_3') &= 0 \end{aligned} \right\} \quad (15)$$

Solve Eq. (15) for target parameters x_1', y_1' . In result after transformations, we receive the following matrix equality:

$$\begin{bmatrix} x_1' \\ y_1' \end{bmatrix} = \mathbf{I}_2 \times \begin{bmatrix} x_2' \\ y_2' \end{bmatrix} + \mathbf{I}_3 \times \begin{bmatrix} x_3' \\ y_3' \end{bmatrix} \quad (16)$$

These are matrixes of transfer function of the dyad from the point 2 to the point 1 and from the point 3 to the point 1. The components of these matrixes are the following:

$$\mathbf{I}_2 = \begin{bmatrix} i2_{11} & i2_{12} \\ i2_{21} & i2_{22} \end{bmatrix}, \quad \mathbf{I}_3 = \begin{bmatrix} i3_{11} & i3_{12} \\ i3_{21} & i3_{22} \end{bmatrix},$$

$$i2_{11} = \frac{\tan \phi_{31}}{\tan \phi_{31} - \tan \phi_{21}}, \quad i2_{12} = \frac{\tan \phi_{21} \cdot \tan \phi_{31}}{\tan \phi_{31} - \tan \phi_{21}},$$

$$i2_{21} = \frac{-1}{\tan \phi_{31} - \tan \phi_{21}}, \quad i2_{22} = \frac{-\tan \phi_{21}}{\tan \phi_{31} - \tan \phi_{21}},$$

$$i3_{11} = \frac{-\tan \phi_{21}}{\tan \phi_{31} - \tan \phi_{21}}, \quad i3_{12} = \frac{-\tan \phi_{21} \cdot \tan \phi_{31}}{\tan \phi_{31} - \tan \phi_{21}},$$

$$i3_{21} = \frac{1}{\tan \phi_{31} - \tan \phi_{21}}, \quad i3_{22} = \frac{\tan \phi_{31}}{\tan \phi_{31} - \tan \phi_{21}}.$$

Having at disposal such transfer functions, it is possible to construct transfer function for any SG or lever mechanism. For example, if SG graph is available then its initial graph is certain Fig. (15).

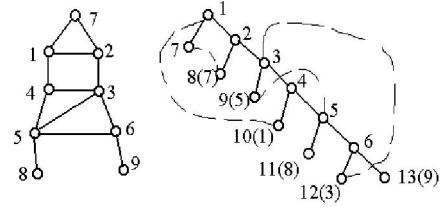


Figure 15: Scheme of SG for definition its transfer function.

Write down for each dyad the equation for transfer function as Eq. (16). In result we receive the system of the equations Eq. (17) describing transfer function of SG.

$$\left. \begin{aligned} \begin{bmatrix} x_6' \\ y_6' \end{bmatrix} &= \mathbf{I}_{12} \times \begin{bmatrix} x_3' \\ y_3' \end{bmatrix} + \mathbf{I}_{13} \times \begin{bmatrix} x_9' \\ y_9' \end{bmatrix} \\ \begin{bmatrix} x_5' \\ y_5' \end{bmatrix} &= \mathbf{I}_{11} \times \begin{bmatrix} x_8' \\ y_8' \end{bmatrix} + \mathbf{I}_6 \times \begin{bmatrix} x_6' \\ y_6' \end{bmatrix} \\ \begin{bmatrix} x_4' \\ y_4' \end{bmatrix} &= \mathbf{I}_{10} \times \begin{bmatrix} x_1' \\ y_1' \end{bmatrix} + \mathbf{I}_5 \times \begin{bmatrix} x_5' \\ y_5' \end{bmatrix} \\ \begin{bmatrix} x_6' \\ y_6' \end{bmatrix} &= \mathbf{I}_{12} \times \begin{bmatrix} x_3' \\ y_3' \end{bmatrix} + \mathbf{I}_{13} \times \begin{bmatrix} x_9' \\ y_9' \end{bmatrix} \\ \begin{bmatrix} x_5' \\ y_5' \end{bmatrix} &= \mathbf{I}_{11} \times \begin{bmatrix} x_8' \\ y_8' \end{bmatrix} + \mathbf{I}_6 \times \begin{bmatrix} x_6' \\ y_6' \end{bmatrix} \\ \begin{bmatrix} x_4' \\ y_4' \end{bmatrix} &= \mathbf{I}_{10} \times \begin{bmatrix} x_1' \\ y_1' \end{bmatrix} + \mathbf{I}_5 \times \begin{bmatrix} x_5' \\ y_5' \end{bmatrix} \end{aligned} \right\} \quad (17)$$

Similar equations may be applying for kinematical analysis and synthesis of lever mechanisms.

7 Conclusions

The stated method of structural synthesis based on revealed general structural properties of the kinematic chains, essentially allows constructing all possible SG. However, this problem has infinitely many decisions and cannot be end in itself.

Kinematical properties of SG are closely connecting with their structure.

The established law of a structure of kinematic chains gives the basis to put the next questions. At first, on what defines number of the dyads for reproduction demanded movement? At second, on what defines the rational organization of feedbacks between dyads for the reproduction demanded movement? Answers for these questions would enable to make purposeful structural synthesis of the kinematic chains for concrete conditions of movement.

Thus, the main advantage of a method is that any Asura group can be built of the same-type elements - diads that gives additional opportunities for their analysis and a synthesis.

References

- [1] Johnson, R.C., 1965, "Application of Number Synthesis to Practical Problems in Creative Design," ASME Paper No. 65-WA-MD9.
- [2] Crossley, F.R.E., 1965, "The Permutations of Kinematic Chains of Eight Member or Less from the Graph-Theoretic Viewpoint." *Developments in Theoretical and Applied Mechanisms*, Vol.2, Ed. by W.A. Shaw, Pergamon Press, Oxford, pp. 467-489.
- [3] Freudenstein, F. and Maki, E.R., 1979, "The Creation of Mechanisms according to Kinematic Structure and Function," *Journal of Environment and Planning B*, Vol. 6, pp. 375-391.
- [4] Yan, H.S., 1992, "A Methodology for Creative Mechanism Design," *Mechanism and Machine Theory*, Vol. 27, pp. 235-242.
- [5] Freudenstein, F., 1967, "The Basic Concepts of Polya's Theory of Enumeration with Application to the Structural Classification of Mechanisms," *ASME Journal of Mechanisms*, Vol. 3, pp. 275-290.
- [6] Bushsbaum, F. and Freudenstein, F., 1970, "Synthesis of Kinematic Structure of Geared Kinematic Chains and other Mechanisms," *ASME Journal of Mechanisms*, Vol. 5, pp. 357-392.
- [7] Yan, H.S. and Hwang, Y.W., 1991, "The Specialization of Mechanisms," *Mechanism and Machine Theory*, Vol. 26, pp. 541-551.
- [8] Yan, H.S. and Hung, C.C., 2006, "Identifying and Counting the Number of Mechanisms from Kinematic Chains subject to Design Constraints," *ASME Journal of Mechanical Design*, Vol. 128, pp. 1177-1182.
- [9] Yan, H.S. and Hung, C.C., 2006, "An Improved Approach for Counting the Number of Mechanisms from Kinematic Chains subject to Design Constraints," *Journal of the Chinese Society of Mechanical Engineers*, Taipei. (Submitted).
- [10] Yan, H.S. and Hung, C.C. "A Systematic Procedure to Count the Number of Mechanisms with Design

Constraints", Proc. 12th IFToMM World Congress, Besancon, 2007.

[11] N.N. Krokmal, *Strukturny analiz e sintez group Asura*, *Izvestia vuzov, Mashinostroenie*, №2 2002, pp.24-30.

[12] F. Harary, *Graph Theory*, Addison-Wesley Publishing Company, London, 1969.

[13] W. Lipski, *Kombinatoryka Dla Programistow, Wydawnictwa Naukowo-Techniczne*, Warszawa, 1982.

[14] E. Peisach, H. Dresig, J. Schonherr. *Typ- und Masssythese von ebenen Koppelgetrieben mit hoeheren Gliedgruppen (Zwischenbericht zum Fortsetzungsantrag)*. - DFG - Themenummer: Dr 234/7-1, TU Chemnitz, Professur Maschinendynamik / Schwingunglehre, Professur Getriebelehre, Chemnitz, 1998.

[15] J.P. Kukluk, L.B.Holder and D.J.Cook, "Algorithm and Experiments in Testing Planar Graphs for Isomorphism," *Journal of Graph Algorithms and Applications*, vol. 8, No. 3, 2004, pp.313-356.

[16] S.N. Kozhevnikov, *Osnovania strukturnogo sinteza mekhanizmov*, *Naukova duomka*, Kiev, 1979.

